Algebra II, Spring 2017

Problem Set 9

Due: April 27 in class

In the following questions, **k** *denotes an algebraically closed field.*

1. (a) Show that if $f \in \mathbf{k}[x_1, \ldots, , x_n]$ is an irreducible polynomial then $V(f)$ is an irreducible algebraic subset.

(b) Show that $f(x,y) = (x^2 - 1)^2 + y^2 \in \mathbf{R}[x,y]$ is irreducible, but $V(f)$ is not irreducible.

2. Prove that if $I \subset \mathbf{k}[x_1, \ldots, x_n]$ is an ideal, then $\sqrt{I}$ is the intersection of all the maximal ideals containing I. (use Hilbert's Nullstellensatz.)

3. Let V be a closed algebraic subset of $\mathbf{A}^n_{\mathbf{k}}$.

(b) Show that every descending chain of closed subsets of V stabilizes.

(b) Show that every open covering of V has a finite subcover.

4. Write the closed algebraic set $x^2 - yz = xz - x = 0$ in $\mathbf{A}^3_{\mathbf{k}}$ as the union of irreducible algebraic sets.

5. If X_1 and X_2 are closed algebraic subsets of $\mathbf{A}^n_{\mathbf{k}}$, then show that

(a) $I(X_1 \cup X_2) = I(X_1) \cap I(X_2)$

(b) $I(X_1 \cap X_2) = \sqrt{I(X_1) + I(X_2)}$

6. Find the ideal of $\mathbf{k}[x,y]$ corresponding to the union of the x-axis and the point $(1,1)$ in $\mathbf{A}^2_{\mathbf{k}}$.