WeBWorK assignment number set12 is due :

The

(* replace with url for the course home page *)

for the course contains the syllabus, grading policy and other information.

This file is /conf/snippets/setHeader.pg you can use it as a model for creating files which introduce each problem set.

The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA's or your professor for help. Don't spend a lot of time guessing – it's not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $sin(3*pi/2)$instead of -1, $e \wedge (ln(2))$ instead of 2, $(2+tan(3))*(4-sin(5)) \wedge 6-7/8$ instead of 27620.3413, etc. Here's the **list of the functions** which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

From Rogawski ET section 16.2, exercise 21.

1. Compute the line integral of the scalar function $f(x,y,z) = 2x^2+8z$
over the curve $\mathbf{c}(t) = (e^t, t^2, t), \quad 0 \le t \le 6$

$\int_C f(x,y,z)\,ds =$ ________

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2. **(1 pt) Library/ASU-topics/setCalculus/stef/stef16_2p2.pg**

Evaluate the line integral $\int_C 7xy^4\,ds$, where C is the right half of the circle $x^2+y^2=4$.

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3. **(1 pt) Library/Rochester/setVectorCalculus1/UR_VC_11_8.pg**

Consider a wire in the shape of a helix $\mathbf{r}(t) = 1\cos t\mathbf{i} + 1\sin t\mathbf{j} + 5t\mathbf{k}, 0 \le t \le 2\pi$ with constant density function $\rho(x,y,z) = 1$.

A. Determine the mass of the wire: ________

B. Determine the coordinates of the center of mass: (____, ____, ____)

C. Determine the moment of inertia about the z-axis: ________

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4. **(1 pt) Library/Rochester/setVectorCalculus1/UR_VC_11_5.pg**

Let C be the curve which is the union of two line segments, the first going from (0, 0) to (-1, -3) and the second going from (-1, -3) to (-2, 0).

Computer the line integral $\int_C -1dy+3dx$.

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5. **(1 pt) Library/Rochester/setVectorCalculus2/ur_vc_12_3.pg**

Suppose C is any curve from $(0,0,0)$ to $(1,1,1)$ and $\mathbf{F}(x,y,z) = (1z+3y)\mathbf{i}+(2z+3x)\mathbf{j}+(2y+1x)\mathbf{k}$. Compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

____

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6. **(1 pt) Library/Rochester/setVectorCalculus1/UR_VC_11_9.pg**

Find the work done by the force field $\mathbf{F}(x,y,z) = 2x\mathbf{i}+2y\mathbf{j}+3\mathbf{k}$ on a particle that moves along the helix $\mathbf{r}(t) = 2\cos(t)\mathbf{i}+2\sin(t)\mathbf{j}+4t\mathbf{k}, 0 \le t \le 2\pi$. ________

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7. **(1 pt) Library/Rochester/setVectorCalculus2/ur_vc_12_15.pg**

Suppose $\mathbf{F} = \mathbf{F}(x,y,z)$ is a gradient field with $\mathbf{F} = \nabla f$, S is a level surface of f, and C is a curve on S. What is the value of the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$?

____

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8. **(1 pt) Library/Dartmouth/setMTWCh6S1/problem_5.pg**

Suppose that $\nabla f(x,y,z) = 2xyze^{x^2}\mathbf{i}+ze^{x^2}\mathbf{j}+ye^{x^2}\mathbf{k}$.

If $f(0,0,0) = -3$, find $f(3,3,3)$.

Hint: As a first step, define a path from (0,0,0) to (3, 3, 3) and compute a line integral.

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9. **(1 pt) set12/pr12.pg**

Consider the vector field $\mathbf{F} = (x^2+y^2, 5xy)$. Compute the line integrals $\int_{\mathbf{c}_1}\mathbf{F}\cdot d\mathbf{r}$ and $\int_{\mathbf{c}_2}\mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{c}_1(t) = (t,t^2)$ and $\mathbf{c}_2(t) = (t,t)$ for $0 \le t \le 1$. Can you decide from your answers whether or not $\mathbf{F}$ is a gradient vector field? Why or why not?

$\int_{\mathbf{c}_1}\mathbf{F}\cdot d\mathbf{r} =$

____

$\int_{\mathbf{c}_2}\mathbf{F}\cdot d\mathbf{r} =$

____

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10. **(1 pt) Library/272/setStewart16_3/ur_vc_12_5.pg**

Determine whether the given set is open, connected, and simply connected. For example, if it is open, connected, but not simply connected, type "YYN" standing for "Yes, Yes, No."

A. $\{(x,y)\,|x>1,y<2\}$

____ B. $\{(x,y)\,|2x^2+y^2<1\}$

____ C. $\{(x,y)\,|x^2-y^2<1\}$

____ D. $\{(x,y)\,|x^2-y^2>1\}$

____ E. $\{(x,y)\,|1<x^2+y^2<4\}$

____

11. **(1 pt) Library/ASU-topics/setCalculus/stef16_3p3.pg**
Consider the vector field $\mathbf{F}(x,y,z)=(2z+2y)\mathbf{i}+(4z+2x)\mathbf{j}+(4y+2x)\mathbf{k}$.

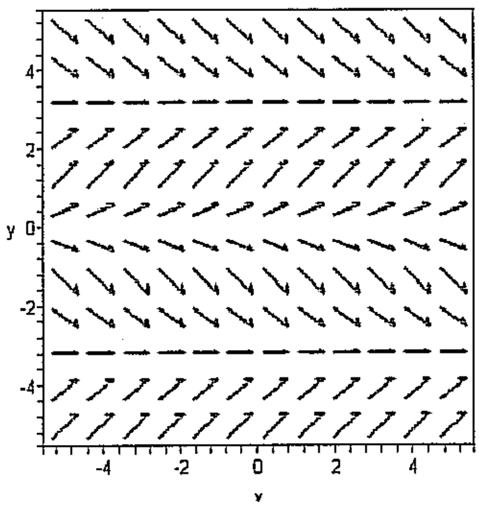
a) Find a function f such that $\mathbf{F}=\nabla f$ and $f(0,0,0)=0$.
$f(x,y,z)=$ ____________

b) Suppose C is any curve from $(0,0,0)$ to $(1,1,1)$. Use part a) to compute the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$.

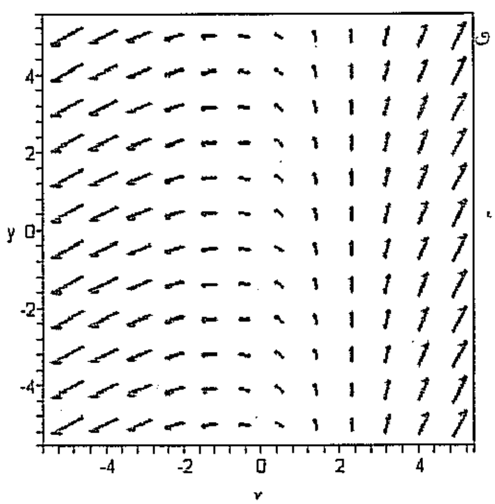
____

12. **(1 pt) Library/ASU-topics/setCalculus/stef/stef16_1p1-/stef16_1p1.pg**

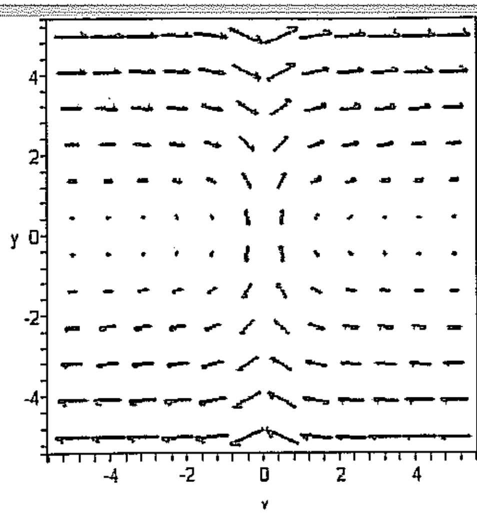
Match the plots labeled A - D with the vector fields $\mathbf{F}$ below.



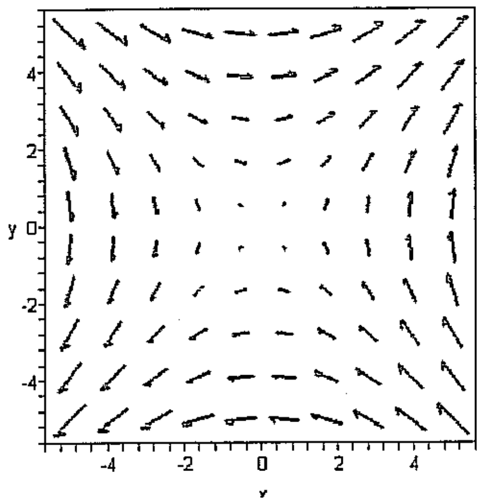
A.



B.



C.



D.

C 1. $\mathbf{F}=\langle y,1/x\rangle$
B 2. $\mathbf{F}=\langle x-2,x+1\rangle$
D 3. $\mathbf{F}=\langle y,x\rangle$
A 4. $\mathbf{F}=\langle 1,\sin y\rangle$

A: the vector field does not depend on x: for fixed y_0 the vector at every point (x,y_0) is the same. so the 4th vector field corresponds to A

B: the vector field does not depend on y, so the second vector field corresponds to B.

1. $\vec{r}(t) = \langle e^t, t^2, t \rangle \qquad 0 \le t \le 6$

$\vec{V}(t) = \langle e^t, 2t, 1 \rangle$

$|\vec{V}(t)| = \sqrt{e^{2t} + 4t^2 + 1}$

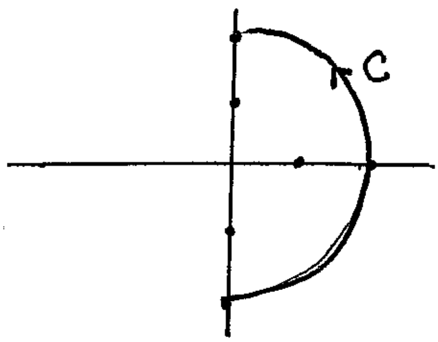
$f(x,y,z) = 2x^2 + 8z = 2e^{2t} + 8t$

$$\int_C f(x,y,z)\, ds = \int_0^6 (2e^{2t} + 8t)\sqrt{e^{2t} + 4t^2 + 1}\ dt$$

$$= \frac{2}{3}\left(e^{2t} + 4t^2 + 1\right)^{\frac{3}{2}} \Big|_0^6$$

$$= \frac{2}{3}\left(e^{12} + 144 + 1\right)^{\frac{3}{2}} - \frac{2}{3}$$

2.



$\vec{r}(t) = \langle 2\cos t, 2\sin t \rangle \qquad -\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$\vec{v}(t) = \langle -2\sin t, 2\cos t \rangle$

$|\vec{V}(t)| = 2$

$7xy^4 = 7(2\cos t)(2\sin t)^4 = 224 \cos t \sin^4 t$

$$\int_C 7xy^4\, ds = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (224 \cos t \sin^4 t)\, 2\, dt$$

$$= \frac{448}{5} \sin^5 t \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{896}{5}$$

5. A. Mass $= \int_C \delta(x,y,z)\, ds = \int_C 1\, ds$

$\vec{r}(t) = \langle \cos t, \sin t, 5t \rangle \quad 0 \le t \le 2\pi$

$\vec{v}(t) = \langle -\sin t, \cos t, 5 \rangle$

$|\vec{v}(t)| = \sqrt{1+25} = \sqrt{26}$

δ Mass $= \int_C 1\, ds = \int_0^{2\pi} 1 \cdot |\vec{v}(t)|\, dt = \int_0^{2\pi} \sqrt{26}\, dt$

$= \boxed{2\sqrt{26}\,\pi}$

B. $\int_C x\rho\, ds = \int_0^{2\pi} (\cos t)\sqrt{26}\, dt = \sqrt{26} \sin t \Big|_0^{2\pi} = 0$

$x = \cos t$
$\rho = 1$

$\int_C y\rho\, ds = \int_0^{2\pi} \sin t \sqrt{26}\, dt = -\sqrt{26} \cos t \Big|_0^{2\pi} = 0$

$\int_C z\rho\, ds = \int_0^{2\pi} 5t\sqrt{26}\, dt = 5\sqrt{26}\, \frac{t^2}{2} \Big|_0^{2\pi} = 10\sqrt{26}\,\pi^2$

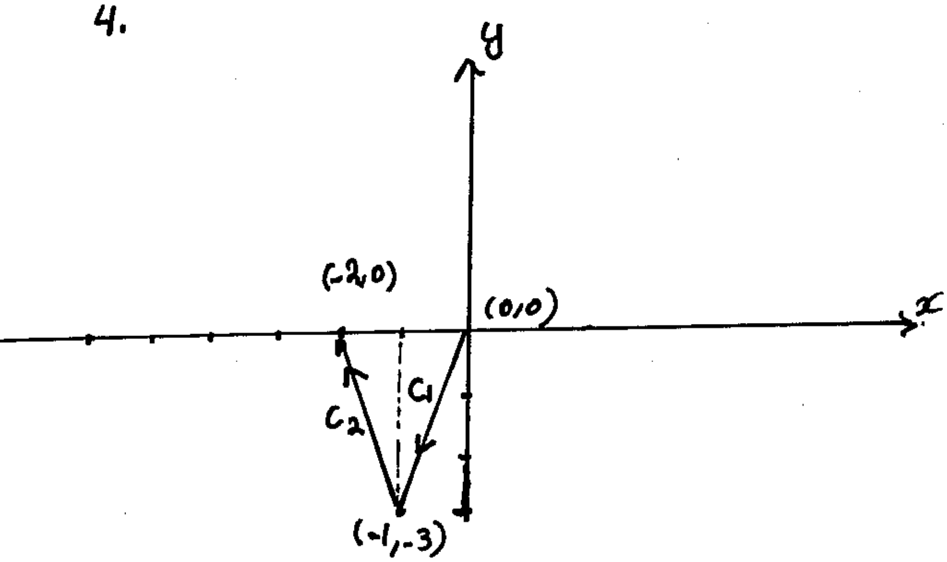
There fore

$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{0}{\text{Mass}}, \frac{0}{\text{Mass}}, \frac{10\sqrt{26}\,\pi^2}{\text{Mass}} \right) = \boxed{(0, 0, 5\pi)}$

C. $I_z = \int_C (x^2+y^2)\,\rho\, ds = \int_0^{2\pi} (\cos^2 t + \sin^2 t) \cdot 1 \cdot \sqrt{26}\, dt$

$$= \int_0^{2\pi} \sqrt{26}\ dt = \boxed{2\sqrt{26}\ \pi}$$

4.



$C_1:\quad \vec{r}(t) = \langle -t, -3t \rangle \quad 0 \le t \le 1$

$\quad\quad \vec{v}(t) = \langle -1, -3 \rangle$, so $|\vec{v}(t)| = \sqrt{10}$

$\vec{F} = \langle 3, -1 \rangle$

$$\int_C -1\,dy + 3\,dx = \int_{C_1} -1\,dy + 3\,dx + \int_{C_2} -1\,dy + 3\,dx$$

$$\int_{C_1} -dy + 3\,dx = \int_{C_1} \vec{F} \cdot d\vec{r} = \int_0^1 \langle 3, -1 \rangle \cdot \langle -1, -3 \rangle\ dt$$

$$= \int_0^1 -3 + 3\ dt = 0$$

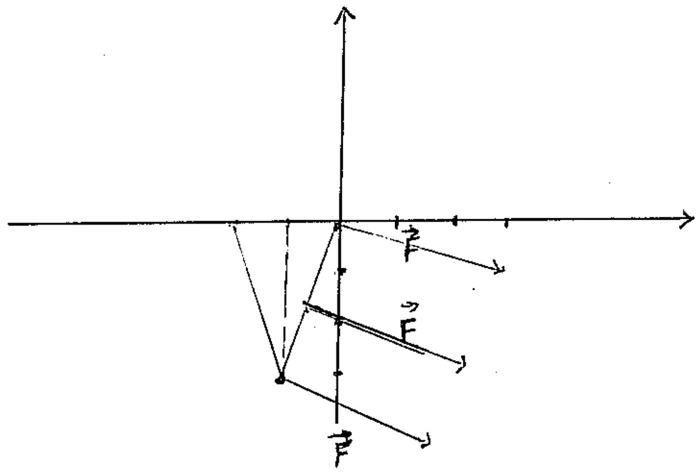
$C_2:\quad \vec{r}(t) = \langle -t-1, 3t-3 \rangle \quad\quad 0 \le t \le 1$

$\quad\quad \vec{v}(t) = \langle -1, 3 \rangle$

$$\int_{C_2} -dy + 3\,dx = \int_{C_2} \vec{F} \cdot d\vec{r} = \int_0^1 \langle 3, -1 \rangle \cdot \langle -1, 3 \rangle\ dt = \int_0^1 -6\ dt = -6$$

therefore $\int_{C} -dy + 3\,dx = -6 + (-6) = -6$

(In this question, the vector field $\vec{F} = \langle 3, -1 \rangle$ is a constant vector field.



$\vec{F}$ is alway perpendicular to the curve C_1 and that is why

$\int_{C_1} \vec{F} \cdot d\vec{r} = \int_{C_1} \vec{F} \cdot \vec{T}\, ds = \int_{C_1} 0 = 0.$

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5. $\vec{F} = \langle \underbrace{z + 3y}_{M}, \underbrace{2z + 3x}_{N}, \underbrace{2y + x}_{P} \rangle$

($M_y = 3 = N_x$ $\quad M_z = 1 = P_x$ $\quad N_z = 2 = P_y$) **

therefore $\vec{F}$ is conservative and $\int_C \vec{F} \cdot dr$ is path independent. We could take the straigh line segment

$\vec{r}(t) = \langle t, t, t \rangle \quad 0 \le t \le 1$

$\vec{v}(t) = \langle 1, 1, 1 \rangle$

$\vec{F} = \langle 4t, 5t, 3t \rangle$ on C, therefore

$$\int_C \vec{F}\cdot dr = \int_0^1 \langle 4t, 5t, 3t\rangle \cdot \langle 1, 1, 1\rangle\, dt = \int_0^1 12t\, dt$$

$$= 6$$

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6. $\vec{r}(t) = \langle 2\cos t, 2\sin t, 4t\rangle \quad 0 \le t \le 2\pi$
   $\vec{v}(t) = \langle -2\sin t, 2\cos t, 4\rangle$

$\vec{F} = \langle 2x, 2y, 3\rangle = \langle 4\cos t, 4\sin t, 3\rangle$

therefore

$$W = \int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \langle 4\cos t, 4\sin t, 3\rangle \cdot \langle -2\sin t, 2\cos t, 4\rangle\, d$$

$$= \int_0^{2\pi} -8\cos t\sin t + 8\cos t\sin t + 12 \quad dt$$

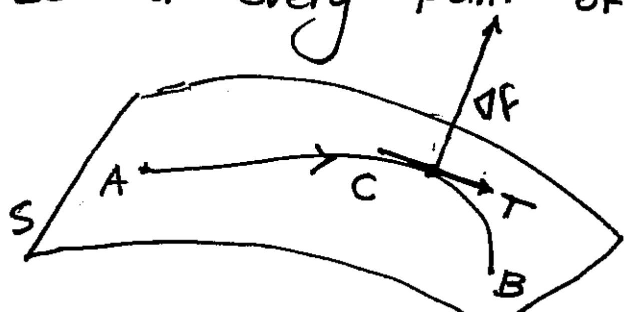
$$= \int_0^{2\pi} 12 \quad dt = 24\pi$$

---

7. $\int_C \vec{F}\cdot dr = \int_C \vec{F}\cdot\vec{T}\, ds$ where $T = \frac{v}{|v|}$ is the unit tangent vector.

Since C is on S, and since S is a level surface for f, ∇f is perpendicular to S and so perpendicular to C. Therefor $\nabla f \cdot T = 0$ at every point of C.

so $\int_C \vec{F}\cdot\vec{T}\, ds = \int_C 0 \quad ds = 0$



8. Let C be a line segment from $(0,0,0)$ to $(3,3,3)$.

Since, by the fundamental theorem of line integrals

$$\int_C \nabla f \cdot d\vec{r} = f(3,3,3) - f(0,0,0), \quad \text{we have}$$

$$f(3,3,3) = \int_C \nabla f \cdot d\vec{r} + f(0,0,0) = \int_C \nabla f \cdot d\vec{r} - 3 \qquad (*)$$

to compute the line integral, we parametrize C:

$$C: \quad \vec{r}(t) = \langle t, t, t \rangle \qquad 0 \le t \le 3$$

$$\vec{v}(t) = \langle 1, 1, 1 \rangle$$

$$\nabla f = \langle 2xyze^{x^2}, ze^{x^2}, ye^{x^2} \rangle = \langle 2t^3e^{t^2}, te^{t^2}, te^{t^2} \rangle.$$

Therefore,

$$\int_C \nabla f \cdot d\vec{r} = \int_0^3 \langle 2t^3e^{t^2}, te^{t^2}, te^{t^2} \rangle \cdot \langle 1,1,1 \rangle \, dt = \int_0^3 2t^3e^{t^2} + 2te^{t^2} \, dt$$

$$= \int_0^3 2(t^3+t)e^{t^2} \, dt \underset{\uparrow}{=} (t^2+1)e^{t^2}\Big|_0^3 - \int_0^3 2te^{t^2} \, dt$$

integration by part

$$= (10e^9 - 1) - e^{t^2}\Big|_0^3 = 10e^9 - 1 - e^9 + 1 = 9e^9$$

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9. $C_1: \vec{r}(t) = \langle t, t^2 \rangle, \quad \vec{v}(t) = \langle 1, 2t \rangle$

$$\vec{F} = \langle x^2+y^2, 5xy \rangle = \langle t^2 + t^4, 5t^3 \rangle$$

So

$$\int_{C_1} \vec{F}\cdot d\vec{r} = \int_0^1 \langle t^2+t^4, 5t^3\rangle \cdot \langle 1, 2t\rangle \, dt = \int_0^1 (t^2+t^4+10t^4)\, dt$$

$$= \frac{t^3}{3} + \frac{t^5}{5} + 2t^5 \Big|_0^1 = \frac{1}{3} + \frac{1}{5} + 2 = \frac{38}{15}$$

C_2: $\quad \vec{r}(t) = \langle t, t\rangle \qquad \vec{v}(t) = \langle 1, 1\rangle$

$\vec{F} = \langle x^2+y^2, 5xy\rangle = \langle 2t^2, 5t^2\rangle$

So

$$\int_{C_2} \vec{F}\cdot d\vec{r} = \int_0^1 \langle 2t^2, 5t^2\rangle \cdot \langle 1, 1\rangle \, dt = \int_0^1 7t^2 \, dt$$

$$= \frac{7}{3} t^3 \Big|_0^1 = \frac{7}{3}.$$

$\int_{C_1} \vec{F}\cdot d\vec{r} \neq \int_{C_2} \vec{F}\cdot d\vec{r}$, therefor $\vec{F}$ is not a conservative vector field

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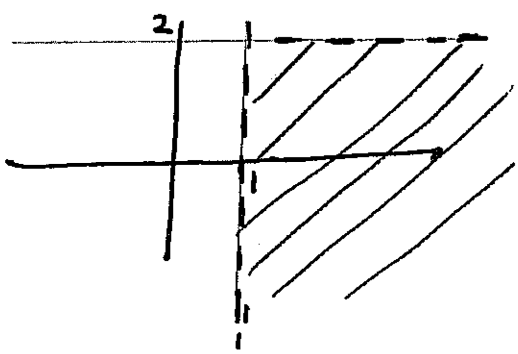
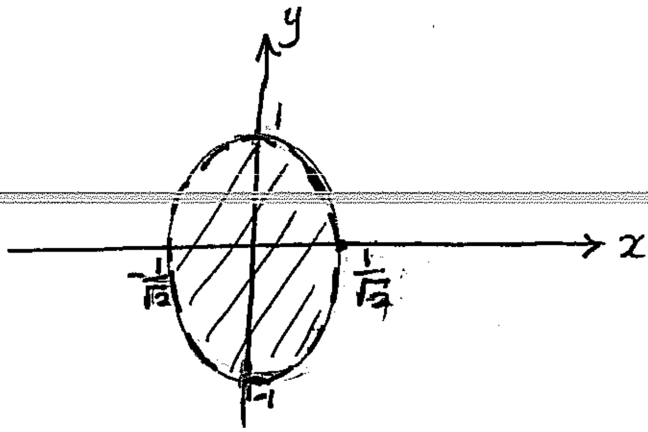
10.

A:



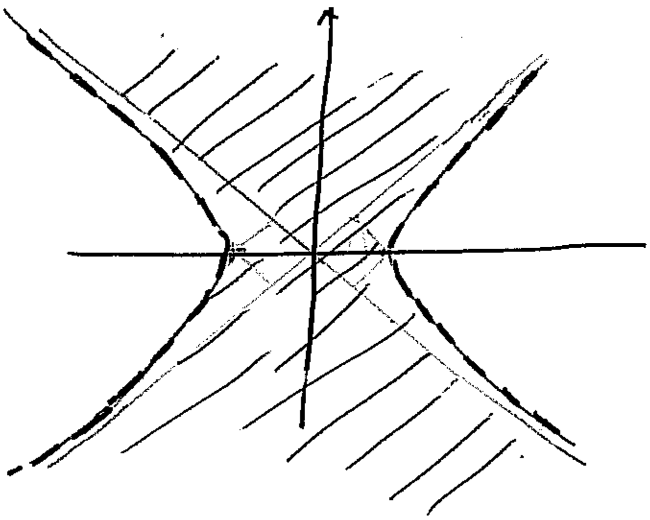
the region is open, connected, and simply connected

B.



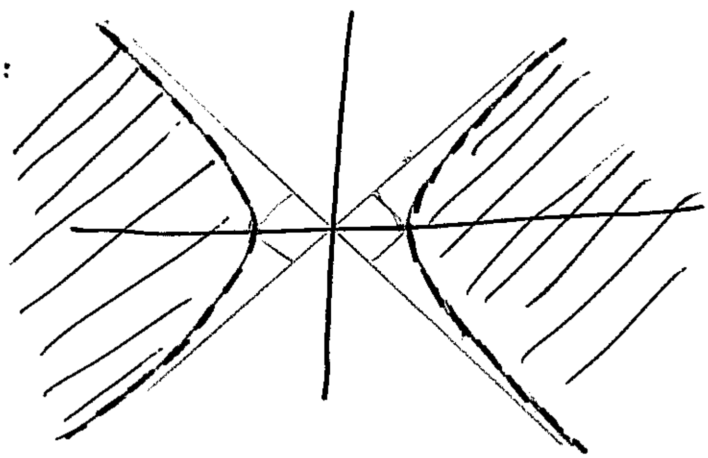
the region is open, connected, and simply connected.

C:



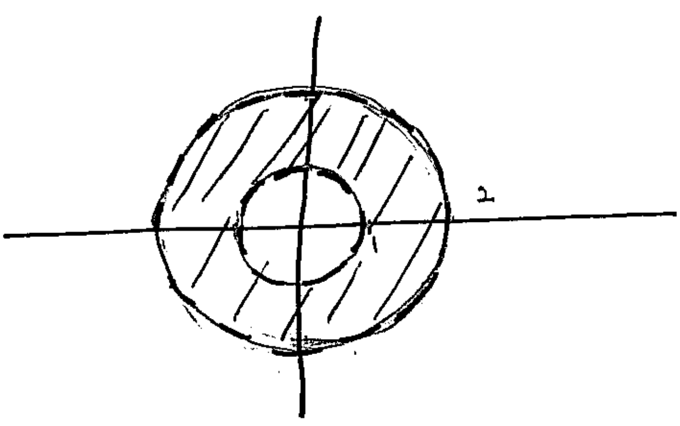
the region is open connected and simply connected

D:



the region is open, but not connected (points A and B cannot be connected by a curve which lies completely in the region)
since the region is not connected, it's not simply connected either

E:



the region is open and connected, but not simply connected.

11.

(a)

$$\nabla f = \langle f_x, f_y, f_z \rangle = \langle 2z+2y,\ 4z+2x,\ 4y+2x \rangle$$

$$f = \int (2z+2y)\,dx = 2zx + 2yx + g(y,z)$$

$$f_y = 4z+2x \Rightarrow \frac{\partial}{\partial y}(2zx+2yx+g(y,z)) = 4z+2x$$

$$\Rightarrow \cancel{2x} + g_y = 4z + \cancel{2x} \Rightarrow g_y = 4z \Rightarrow g = \int 4z\,dy$$

$$\Rightarrow g = 4zy + k(z)$$

So $f = 2zx + 2yx + 4zy + k(z)$

$$f_z = 4y+2x \Rightarrow \frac{\partial}{\partial z}(2zx+2yx+4zy+k(z)) = 4y+2x$$

$\Rightarrow 2x + 4y + k'(z) = 4y + 2x \Rightarrow k'(z) = 0 \Rightarrow k(z) = C$ a constant

$\Rightarrow$ any function of the form $2zx + 2yx + 4zy +$ constant is a potential function for $\vec{F}$, since we want $f(0,0,0)=0$, the constant is zero, so $f = 2zx + 2yx + 4zy$.

(In this problem, it is easier to guess what the potential function looks like instead of integrating)

(b). $$\int_C F \cdot d\vec{r} = \int_C \nabla f \cdot d\vec{r} = f(1,1,1) - f(0,0,0)$$

$$= (2+2+4) - 0 = 8$$

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12. 1. C 2. B 3. D 4. A