Roya Beheshti Zavareh **math233fall2008**

WeBWorK assignment number set14 is due : 12/11/2008 at 10:00pm CST.

The

(* replace with url for the course home page *)

for the course contains the syllabus, grading policy and other information.

This file is /conf/snippets/setHeader.pg you can use it as a model for creating files which introduce each problem set.

The primary purpose of WeBWorK is to let you know that you are getting the correct answer or to alert you if you are making some kind of mistake. Usually you can attempt a problem as many times as you want before the due date. However, if you are having trouble figuring out your error, you should consult the book, or ask a fellow student, one of the TA's or your professor for help. Don't spend a lot of time guessing – it's not very efficient or effective.

Give 4 or 5 significant digits for (floating point) numerical answers. For most problems when entering numerical answers, you can if you wish enter elementary expressions such as $2 \wedge 3$ instead of 8, $sin(3*pi/2)$instead of -1, $e \wedge (ln(2))$ instead of 2, $(2+tan(3))*(4-sin(5))\wedge 6-7/8$ instead of 27620.3413, etc. Here's the **list of the functions** which WeBWorK understands.

You can use the Feedback button on each problem page to send e-mail to the professors.

From Rogawski ET section 16.4, exercise 23.

Calculate $\iint_S f(x,y,z)\,dS$ For

$$x^2+y^2=25, \quad 0 \le z \le 4; \qquad f(x,y,z)=e^{-z}$$

$\iint_S f(x,y,z)\,dS =$ ________

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2. (1 pt) Library/Dartmouth/setMTWCh6S4/problem_2.pg

Find the flux of $\mathbf{F}(x,y,z) = (3xy^2, 3x^2y, z^3)$ out of the sphere of radius 1 centered at the origin. Hint: Use spherical coordinates and be mindful of the orientation.

The flux is given by the integral:

$1^5 \int_a^b \int_c^d f(\theta,\phi)d\theta\, d\phi$ where:

$a =$____, $b =$____, $c =$____, $d =$____, and

$f(\theta,\phi) =$ ________________

(use variables "t" for theta and "p" for phi).

The value of the integral is ________.

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3. (1 pt) Library/maCalcDB/setVectorCalculus3/ur_vc_13_4.pg

Let S be the part of the plane $4x+2y+z=4$ which lies in the first octant, oriented upward. Find the flux of the vector field $\mathbf{F} = 2\mathbf{i}+3\mathbf{j}+3\mathbf{k}$ across the surface S.

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4. (1 pt) Library/Rochester/setVectorCalculus2/ur_vc_12_1.pg

For each of the following vector fields $\mathbf{F}$, decide whether it is conservative or not by computing curl $\mathbf{F}$. Type in a potential function f (that is, $\nabla f = \mathbf{F}$). If it is not conservative, type N.

A. $\mathbf{F}(x,y) = (-4x+1y)\,\mathbf{i}+(1x+4y)\,\mathbf{j}$

$f(x,y) =$ ________

B. $\mathbf{F}(x,y) = -2y\mathbf{i} - 1x\mathbf{j}$

$f(x,y) =$ ________

C. $\mathbf{F}(x,y,z) = -2x\mathbf{i} - 1y\mathbf{j} + \mathbf{k}$

$f(x,y,z) =$ ________

D. $\mathbf{F}(x,y) = (-2\sin y)\,\mathbf{i}+(2y-2x\cos y)\,\mathbf{j}$

$f(x,y) =$ ________

E. $\mathbf{F}(x,y,z) = -2x^2\mathbf{i}+1y^2\mathbf{j}+2z^2\mathbf{k}$

$f(x,y,z) =$ ________

Note: Your answers should be either expressions of x, y and z (e.g. "3xy + 2yz"), or the letter "N"

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5. (1 pt) Library/272/setStewart16_5/problem_4.pg

Show that the vector field $F(x,y,z) = (-8y\cos(9x), 9x\sin(-8y), 0)$ is not a gradient vector field by computing its curl. How does this show what you intended?

$\text{curl}(F) = \nabla \times F =$ (______, ______, ______).

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6. (1 pt) Library/OSU/accelerated_calculus_and_analytic_geometry_iii-/hmwk8/prob7.pg

Use Stoke's theorem to evaluate $\iint_S curl\mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x,y,z) = -4yz\mathbf{i}+4xz\mathbf{j}+12(x^2+y^2)z\mathbf{k}$ and S is the part of the paraboloid $z=x^2+y^2$ that lies inside the cylinder $x^2+y^2=1$, oriented upward.

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7. (1 pt) Library/OSU/accelerated_calculus_and_analytic_geometry_iii-/hmwk8/prob8.pg

Use Stoke's Theorem to evaluate $\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}(x,y,z) = x\mathbf{i}+y\mathbf{j}+2(x^2+y^2)\mathbf{k}$ and C is the boundary of the part of the paraboloid where $z=36-x^2-y^2$ which lies above the xy-plane and C is oriented counterclockwise when viewed from above.

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8. (1 pt) Library/Dartmouth/setMTWCh7S2/problem_1.pg

Let $\mathbf{F} = (2x, 2y, 2x+2z)$.

Use Stokes' theorem to evaluate the integral of $\mathbf{F}$ around the curve consisting of the straight lines joining the points (1,0,1), (0,1,0) and (0,0,1).

In particular, compute the unit normal vector and the curl of $\mathbf{F}$ as well as the value of the integral:

$\mathbf{n} =$ (______, ______, ______) (the unit normal vector)

$\nabla \times \mathbf{F} =$ (______, ______, ______)

The value of the integral is ________.

9. (1 pt) Library/Dartmouth/setMTWCh6S2/problem_2.pg

An equation of the tangent plane to the parametrized surface at the point corresponding to $u=-2, v=2$.
$x=3u^2, y=v^2, z=3u^2+v^2$ is (in the variables x, y, z).
To normalize the answer, make sure your coefficient of x is 1.
____________________ $=0$.

10. (1 pt) Library/maCalcDB/setVmultivariable4Linearization-/ur_vc_6_6.pg

Find an equation of the tangent plane to the parametric surface $x=1r\cos\theta$, $y=-5r\sin\theta$, $z=r$ at the point $\left(1\sqrt{2},-5\sqrt{2},2\right)$ when $r=2$, $\theta=\pi/4$.

z = ____________________

Note: Your answer should be an expression of x and y; e.g. "3x - 4y"

1) using cylindrical coordinates. $x = r\cos\theta$ $y = r\sin\theta$ z,
we get a parametrization of S using parameters θ and z

$$r(\theta, z) = \langle 5\cos\theta, 5\sin\theta, z\rangle \qquad 0 \le \theta \le 2\pi \qquad 0 \le z \le 4.$$

$$\vec{r}_\theta = \langle -5\sin\theta, 5\cos\theta, 0\rangle$$
$$\vec{r}_z = \langle 0, 0, 1\rangle$$
$$\vec{r}_\theta \times \vec{r}_z = \langle 5\cos\theta, 5\sin\theta, 0\rangle \Rightarrow |\vec{r}_\theta \times r_z| = 5$$

Therefore $\displaystyle\iint_S f(x,y,z)\, d\sigma = \iint_S e^{-z}\, d\sigma = \int_0^{2\pi}\int_0^4 e^{-z} \times 5 \, dz\, d\theta$

$$= \int_0^{2\pi} -5e^{-z}\Big|_0^4 \, d\theta = \int_0^{2\pi} -5e^{-4} + 5 \, d\theta$$
$$= 10\pi\,(1 - e^{-4})$$

2) A parametrization for S is given by

$$\vec{r}(\phi, \theta) = \langle \sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi\rangle \qquad \begin{matrix} 0 \le \theta \le 2\pi \\ 0 \le \phi \le \pi \end{matrix}$$
$$\vec{r}_\phi = \langle \cos\phi\cos\theta, \cos\phi\sin\theta, -\sin\phi\rangle$$
$$\vec{r}_\theta = \langle -\sin\phi\sin\theta, \sin\phi\cos\theta, 0\rangle$$

$$\vec{r}_\phi \times \vec{r}_\theta = \langle \sin^2\phi\cos\theta, \sin^2\phi\sin\theta, \sin\phi\cos\phi\rangle$$

at $\theta = 0$ $\phi = \frac{\pi}{4}$ for example, we get

$\vec{r}_\phi \times \vec{r}_\theta = \langle \frac{1}{2}, 0, \frac{1}{2}\rangle$ This is pointing toward outside of the sphere, so this is the correct orientation.

we have

$$\text{outward flux of } \vec{F} = \iint_S (\vec{F}\cdot\vec{n})\, d\sigma = \int_0^{\pi}\int_0^{2\pi} \vec{F}\cdot(\vec{r}_\phi \times \vec{r}_\theta)\, d\theta\, d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} \langle 3\sin^3\phi\cos\theta\sin^2\theta,\ 3\sin^3\phi\cos^2\theta\sin\theta,\ \cos^3\phi\rangle \cdot \langle \sin^2\phi\cos\theta,\ \sin^2\phi\sin\theta,\ \sin\phi\cos\phi\rangle \quad d\theta\, d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} 6\sin^5\phi\cos^2\theta\sin^2\theta + \cos^4\phi\sin\phi \quad d\theta\, d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} \frac{3}{2}\sin^5\phi\ \sin^2(2\theta) + \cos^4\phi\ \sin\phi \quad d\theta\, d\phi$$

$$= \int_0^{\pi}\int_0^{2\pi} \frac{3}{2}\sin^5\phi\ \frac{1-\cos 4\theta}{2} + \cos^4\phi\ \sin\phi \quad d\theta\, d\phi$$

$$= \int_0^{\pi} \frac{3}{4}\sin^5\phi \times 2\pi + \cos^4\phi\ \sin\phi \times 2\pi \quad d\phi$$

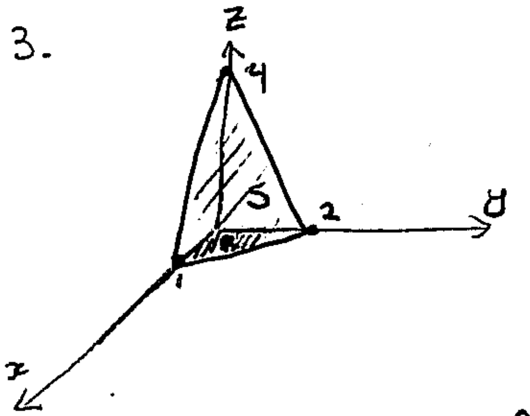
$$= \int_0^{\pi} \frac{3}{2}\pi\ \sin\phi(1-\cos^2\phi)^2 + 2\pi\cos^4\phi\ \sin\phi\ d\phi$$

$$= \int_0^{\pi} \frac{3}{2}\pi\ \sin\phi + \frac{3}{2}\pi\sin\phi\ \cos^4\phi - 3\pi\sin\phi\cos^2\phi + 2\pi\cos^4\phi\sin\phi\ d\phi$$

$$= \left(-\frac{3}{2}\pi\cos\phi - \frac{3}{10}\pi\cos^5\phi + \pi\cos^3\phi - \frac{2\pi}{5}\cos^5\phi\right)\Big|_0^{\pi}$$

$$= \left(\frac{3}{2}\pi + \frac{3}{10}\pi - \pi + \frac{2\pi}{5}\right)\times 2 = \frac{12}{5}\pi$$

3.



A parametrization for S is given by

$\vec{r}(x,y) = \langle x, y, 4-4x-2y \rangle$ (x,y) belongs to R

$\vec{r}_x = \langle 1, 0, -4 \rangle$

$\vec{r}_y = \langle 0, 1, -2 \rangle$

$\vec{r}_x \times \vec{r}_y = \langle 4, 2, 1 \rangle$. this is pointing upward, so we have the correct direction.

flux of $\vec{F}$ outward $= \iint_S (\vec{F} \cdot \vec{n})\, d\sigma = \iint_R \vec{F} \cdot (\vec{r}_x \times \vec{r}_y)\, dA$

$= \iint_R \langle 2,3,3 \rangle \cdot \langle 4,2,1 \rangle\, dA = \iint_R 17\, dA = 17 \iint_R 1\, dA$

$= 17$ area of $R = 17$

4. when a vector field $\vec{F}$ is defined over the whole space (or plane) since the region is simply connected $\vec{F} = \langle M, N, P \rangle$ is conservative exactly when

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \qquad \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x} \qquad \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}.$$

(In the case of ~~a~~ plane ~~a~~ $\vec{F} = \langle M, N \rangle$ which is defined over the whole plane is conservative exactly when $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$)

(A) : $\vec{F}(x,y) = \langle -4x+y,\ x+4y \rangle$

$$\frac{\partial(-4x+y)}{\partial y} = 1 = \frac{\partial(x+4y)}{\partial x}$$

Conservative $f = \cancel{\ \ }\ -2x^2 + yx + 2y^2$

(B) : $\vec{F}(x,y) = -2y\,\vec{i} - x\,\vec{j}$

$$\frac{\partial(-2y)}{\partial y} = -2 \neq -1 = \frac{\partial(-x)}{\partial x}$$

Not conservative

(c) : $\vec{F}$ is conservative. $f = -x^2 - \frac{y^2}{2} + z$

(d) : $\vec{F}$ is conservative $f = y^2 + (-2\sin y)x$

(e) $\vec{F}$ is conservative $f = \frac{-2}{3}x^3 + \frac{y^3}{3} + \frac{2z^3}{3}$

To find the potential function, you can do integration with respect to x, y, and z. (like the example we did in class) or you can simply try to guess what f is.

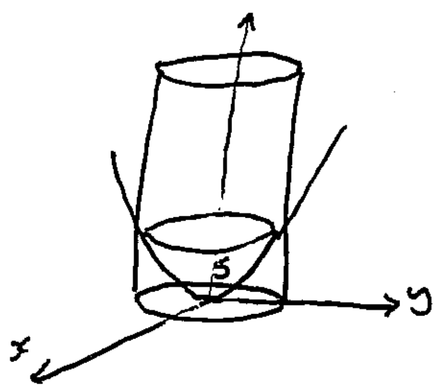
so that it partial derivative matches the ~~given~~ components of the vector field.

5. If $\vec{F} = \langle M, N, P \rangle$ defined over the whole space (a simply connected region) is conservative, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$, $\frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}$, $\frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$, so its curl $= 0$.

In this case $\vec{F} = \langle -8y\cos(9x), 9x\sin(-8y), 0 \rangle$

$\text{curl } \vec{F} = \langle 0, 0, -72x\cos(-8y) + \cancel{72x}\, 8\cos(9x) \rangle$

6.



we are told that S is oriented upward

so this is the same as what we also call inward

If C denotes the boundary of S, to oreint C positively with respect to the inward ~~fluxes~~ orientation of the paraboloid, we should look at the countaclockwise orientation of C.

Since C is the intersection of $x^2+y^2=1$ with

$z = x^2+y^2$, we have $z=1$ so C is a circle of radius 1 in the plane $z=1$.

Stoke's' Theorem says

$$\int_C \vec{F}\cdot d\vec{r} = \iint_S (\text{curl}\vec{F})\cdot\vec{n}\, d\sigma$$

↑ unit normal vector pointing outward

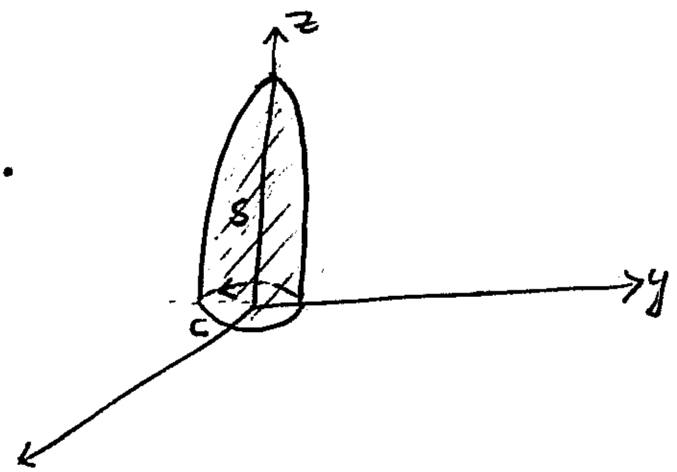
so we compute $\int_C \vec{F}\cdot d\vec{r}$

C is parametrized by

$$\vec{r}(t) = \langle \cos t, +\sin t, 1\rangle \qquad 0\le t\le 2\pi$$
$$\vec{v}(t) = \langle -\sin t, +\cos t, 0\rangle$$

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^{2\pi} \langle 4\sin t, 4\cos t, 12\rangle\cdot\langle -\sin t, +\cos t, 0\rangle\, dt$$
$$= \int_0^{2\pi} +4\sin^2 t + 4\cos^2 t\, dt = \int_0^{2\pi} +4\, dt = +8\pi$$

7.



If we look at the outward orientation of S then the counterclockwise orientation of S is positive.

Stokes' Theorem says

$$\oint_C \vec{F}.d\vec{r} = \iint_S (\text{curl } \vec{F}).\vec{n}\, d\sigma$$

unit normal vector pointing outward.

We compute the right hand side:

A parametrization for S is given by

$$\vec{r}(x,y) = \langle x, y, 36-x^2-y^2 \rangle$$

(x,y) belongs to R: disk of radius 6 on the xy-plane

$$\vec{r}_x = \langle 1, 0, -2x \rangle$$

$$\vec{r}_y = \langle 0, 1, -2y \rangle$$

$$\vec{r}_x \times \vec{r}_y = \langle 2x, 2y, 1 \rangle$$

If we check at one point for example

$x=y=0$ $z=36$ $\vec{r}_x \times \vec{r}_y = \langle 0,0,1 \rangle$ which is pointing outward so this is the correct orientation.

So

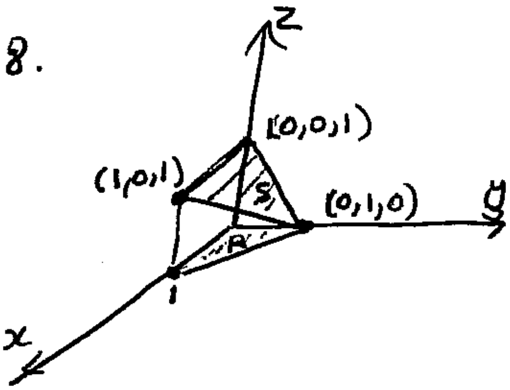
$$\iint_S (\text{curl } \vec{F}) \cdot \vec{n}\, d\sigma = \iint_R \text{curl } \vec{F}.(\vec{r}_x \times \vec{r}_y)\, dA$$

$$= \iint_R \langle 4y, -4x, 0 \rangle \cdot \langle 2x, 2y, 1 \rangle\, dA$$

$$= \iint 8xy - 8xy\, dA = 0$$

$$\left(\vec{F} = \langle x, y, 2x^2+2y^2 \rangle \qquad \text{curl } \vec{F} = \langle 4y, -4x, 0 \rangle \right)$$

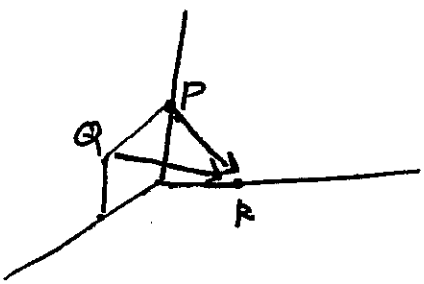
8.



$$\vec{F} = \langle 2x, 2y, 2x+2z \rangle$$

$$\text{curl } \vec{F} = \langle 0, -2, 0 \rangle$$

we find the equation of the plane. and the unit normal vector to the plane.



$\vec{PQ} = \langle 1, 0, 0 \rangle$

$\vec{PR} = \langle 0, 1, -1 \rangle$

$\vec{PQ} \times \vec{PR} = \langle 0, 1, 1 \rangle$

so $\langle 0,1,1 \rangle$ is normal to the plane (and upward)

and so $\vec{n} = \langle 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

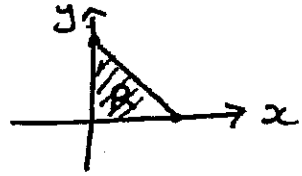
Now that we know the normal vector, we find the equation of the plane:

$$0 \times (x-0) + 1(y-0) + 1(z-1) = 0$$

$$y + z = 0$$

so a parametrization of the part of the plane is given by

$\vec{r}(x,y) = \langle x, y, -y \rangle$ (x,y) belongs to

R: 

$\vec{r}_x = \langle 1, 0, 0 \rangle$

$\vec{r}_y = \langle 0, 1, -1 \rangle$

$\vec{r}_x \times \vec{r}_y = \langle 0, 1, 1 \rangle$

so $\displaystyle\iint_S (\text{curl}\,\vec{F}) \cdot \underbrace{\vec{n}}_{\substack{\text{unit normal}\\ \text{vector upward}}}\, d\sigma = \iint_R \underbrace{\langle 0, -2, 0 \rangle}_{\text{curl}\,\vec{F}} \cdot \underbrace{\langle 0, 1, 1 \rangle}_{\vec{r}_x \times \vec{r}_y}\, dA = \iint_R -2\, dA$

$= -2$ area of Δ

$= -2 \times \frac{1}{2} = -1$

Stokes' Theorem says if we orient C counter clockwise

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S (\text{curl}\,\vec{F}) \cdot \vec{n}\, d\sigma = -1$$

9. $\vec{r}(u,v) = \langle 3u^2, v^2, 3u^2 + v^2 \rangle$

$\vec{r}_u = \langle 6u, 0, 6u \rangle$

$\vec{r}_v = \langle 0, 2v, 2v \rangle$

$\vec{r}_u \times \vec{r}_v = \langle -12uv, -12uv, 12uv \rangle$

$u = -2 \quad v = 2$: $\vec{r}_u \times \vec{r}_v = \langle 48, 48, -48 \rangle$

$r(-2, 2) = \langle 12, 4, 16 \rangle$

since $\vec{r}_u \times \vec{r}_v$ is perpendicular to the tangent plane its equation
r ~~so the~~

at $(12, 4, 16)$ is $48(x-12) + 48(y-4) - 48(z-16) = 0$

dividing by 48: $(x-12)+(y-4)+(z-16)=0$

so $x+y-z=0$

10 $\vec{r}(r,\theta) = \langle r\cos\theta, -5r\sin\theta, r\rangle$

$\vec{r}_r = \langle \cos\theta, -5\sin\theta, 1\rangle$

$\vec{r}_\theta = \langle -r\sin\theta, 5r\cos\theta, 0\rangle$

$\vec{r}_r \times \vec{r}_\theta = \langle 5r\cos\theta, -r\sin\theta, -5r\rangle$

$r=2,\ \theta=\frac{\pi}{4}$ $\vec{r}(2,\frac{\pi}{4}) = \langle \sqrt{2}, -5\sqrt{2}, 2\rangle$

$\vec{r}_r \times \vec{r}_\theta = \langle 5\sqrt{2}, -\sqrt{2}, -10\rangle$

so the equation of the tangent line is:

$$5\sqrt{2}(x-\sqrt{2}) - \sqrt{2}(y+5\sqrt{2}) - 10(z-2) = 0$$

so $5\sqrt{2}x - \sqrt{2}y - 10z = 0$

so $z = \frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{10}y$