

1. What is the angle between the two planes $x + y = 1$ and $2x + y - 2z = 2$?

(a) $\frac{\pi}{2}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$

✓(d) $\frac{\pi}{4}$

(e) The two planes are parallel

(f) $\frac{2\pi}{3}$

$$\vec{u} = \langle 1, 1, 0 \rangle \quad \vec{v} = \langle 2, 1, -2 \rangle \quad \vec{u} \cdot \vec{v} = 3$$

$$\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$$

$$3 = \sqrt{2} \cdot 3 \cdot \cos \theta$$

$$\cos \theta = \frac{\sqrt{2}}{2}$$

$$\theta = \frac{\pi}{4}$$

2

2. If $\mathbf{u} = \langle -8, -2, -4 \rangle$ and $\mathbf{v} = \langle 2, 2, 1 \rangle$, then what is $|\mathbf{u} \times \mathbf{v}|$?

(a) 12

(b) 15

(c) 30

(d) $6\sqrt{2}$

(e) $6\sqrt{3}$

✓(f) $6\sqrt{5}$

$$\vec{u} \times \vec{v} = \begin{pmatrix} i & j & k \\ -8 & -2 & -4 \\ 2 & 2 & 1 \end{pmatrix} = 6i - 12k$$

$$|\vec{u} \times \vec{v}| = \sqrt{6^2 + 12^2} = 6\sqrt{5}$$

3. The rate of change of $f(x, y)$ in the direction of \mathbf{i} is 2 and in the direction of $\frac{\mathbf{i}}{\sqrt{2}} + \frac{\mathbf{j}}{\sqrt{2}}$ is $\sqrt{2}$. In which direction is the rate of change of $f(x, y)$ minimum?

(a) $(1, 0)$

\checkmark (b) $(-1, 0)$

(c) $(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

(d) $(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

(e) $(0, 1)$

(f) $(0, -1)$

$$\nabla f = \langle f_x, f_y \rangle$$

$$\left. \begin{array}{l} \langle f_x, f_y \rangle \cdot \langle 1, 0 \rangle = \sqrt{2} \\ \langle f_x, f_y \rangle \cdot \langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle = \sqrt{2} \end{array} \right\} \Rightarrow \begin{array}{l} f_x = \cancel{\sqrt{2}} \\ \frac{f_x}{\sqrt{2}} + \frac{f_y}{\sqrt{2}} = \sqrt{2}, \text{ so } f_x + f_y = 2 \end{array}$$

$$\text{so } f_y = \frac{0}{2\sqrt{2}} = 0$$

rate of change is minimum in the direction of $-\nabla f = \langle -2, 0 \rangle$

4. At what point is the tangent plane to the surface $y = x^2 + z^2$ parallel to the plane $-x + \frac{y}{2} - 2z = 8$?

✓ (a) $(1, 5, 2)$

(b) $(-1, 5, 2)$

(c) $(1, 5, -2)$

(d) $(\frac{1}{4}, \frac{17}{16}, 1)$

(e) $(\frac{1}{4}, \frac{5}{4}, 1)$

(f) $(-\frac{1}{4}, \frac{17}{16}, 1)$

$$\langle -2x, 1, -2z \rangle = c \langle -1, \frac{1}{2}, -2 \rangle$$

$$\text{so } -1 = \frac{c}{2} \quad \text{so } c = 2.$$

$$\begin{aligned} -2x &= -c = -2, \text{ so } x = 1 \\ -2z &= -2c, \text{ so } z = c = 2 \end{aligned}$$

$$x = 1, z = 2, y = 5$$

5. If $w = z - \sin(xy)$, $x = t$, $y = \ln t$, $z = e^{t-1}$, then what is $\frac{dw}{dt}$ at $t = 1$?

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

(f) $\frac{1}{2}$

$$\begin{aligned}\frac{dw}{dt} &= \frac{\partial w}{\partial x} \frac{dx}{dt} + \frac{\partial w}{\partial y} \frac{dy}{dt} + \frac{\partial w}{\partial z} \frac{dz}{dt} \\ &= -y \cos(xy) + x \frac{\cos(xy)}{t} + e^{t-1}\end{aligned}$$

* $t=1, y=\ln t=0$

$$\begin{aligned}x &= 1 \\ z &= e^{1-1} = 1\end{aligned}$$

so $\frac{dw}{dt} = -1 + 1 = 0$

6. Suppose that $f(x, y)$ is such that $f_x(1, 2) = 4$, $f_y(1, 2) = -1$, $f(1, 2) = 0$. What is the parametric equation of the line of intersection of the plane $y = 2$ and the graph of f at $(1, 2, 0)$?

(a) $x = 1 - t, y = 2, z = 2t$

\checkmark (b) $x = 1 + t, y = 2, z = 4t$

(c) $x = 1 + t, y = 2, z = 1 - t$

(d) $x = 1 - t, y = 2, z = 2t$

(e) $x = 1 - t, y = 2, z = 4t$

(f) $x = 1 - t, y = 2, z = 1 + t$

A vector parallel to the tangent line = $\langle 1, 0, f_x(1, 2) \rangle$
 $= \langle 1, 0, 4 \rangle$

The line passing through $(1, 2, 0)$ Parallel to $\langle 1, 0, 4 \rangle$

$x = 1 + t \quad y = 2 \quad z = 4t$

7. What is the equation of the tangent line to the curve of intersection of surfaces $xyz = 2$ and $3x^2 + y^2 - z^2 = 0$ at $(1, 1, 2)$?

✓ (a) $x = 1 - 5t, \quad y = 1 + 7t, \quad z = 2 - 4t$

(b) $x = 1 + 5t, \quad y = 1 + 7t, \quad z = 2 - 4t$

(c) $x = 1 - 5t, \quad y = 1 - 7t, \quad z = 2 + 4t$

(d) $x = 1 - 5t, \quad y = 1 + 3t, \quad z = 2 + t$

(e) $x = 1 - 5t, \quad y = 1 - 3t, \quad z = 2 + t$

(f) $x = 1 - 5t, \quad y = 1 + 3t, \quad z = 2 - t$

$$\underbrace{xyz - 2 = 0}_{f(x,y,z)} \quad \underbrace{3x^2 + y^2 - z^2 = 0}_{g(x,y,z)}$$

$$\nabla f = \langle yz, xz, xy \rangle = \langle 2, 2, 1 \rangle$$

$$\nabla g = \langle 6x, 2y, -2z \rangle = \langle 6, -2, -4 \rangle$$

$$\langle 2, 2, 1 \rangle \times \langle 6, -2, -4 \rangle = \langle -10, 14, -8 \rangle = 2 \langle -5, 7, -4 \rangle$$

A line passing through $(1, 1, 2)$ parallel to $\langle -5, 7, -4 \rangle$:

$$x = 1 - 5t \quad y = 1 + 7t \quad z = 2 - 4t$$

8. What is the absolute maximum value of the function $f(x, y) = 2x^2 + 3y^2 - 4x - 7$ over the disk $x^2 + y^2 \leq 16$?

(a) -9

(b) 9

(c) 29

(d) 37

(e) 40

\checkmark (f) 45

critical point $4x - 4 = 0$ $(1, 0)$ $f(1, 0) = 2 - 4 - 7 = -9$
 $6y = 0$

on the boundary, we use Lagrange multipliers:

$$4x - 4 = \lambda(2x)$$

$$\Rightarrow 4x - 4 = 6x, \text{ so } x = -2$$

$$6y = \lambda(2y) \Rightarrow \lambda = 3$$

$$\text{so } y^2 = 16 - 4 = 12$$

$$f(x, y) = 2(-2)^2 + 3(12) - 4(-2) - 7 = 8 + 36 + 8 - 7 = 45$$

9. What is the volume of the solid bounded by the planes $z + 2x + y = 4$, $x = 0$, $y = 0$, $z = 0$?

(a) $\frac{4}{3}$

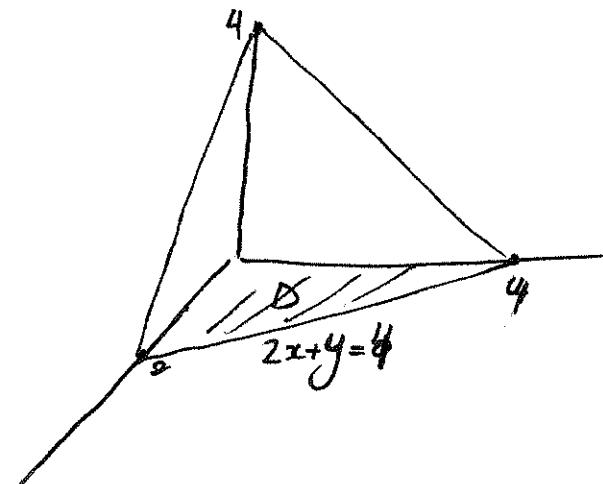
(b) $\frac{1}{3}$

(c) 2

✓ (d) $\frac{16}{3}$

(e) $\frac{8}{3}$

(f) $\frac{7}{6}$



$$\text{volume} = \iint_D 4 - y - 2x \, dA$$

$$= \int_0^2 \int_0^{4-2x} 4 - 2x - y \, dy \, dx$$

$$= \int_0^2 (4 - 2x)y - \frac{y^2}{2} \Big|_{y=0}^{y=4-2x} \, dx = \int_0^2 (4 - 2x)^2 - \frac{(4 - 2x)^2}{2} \, dx$$

$$= \int_0^2 \frac{(4 - 2x)^2}{2} \, dx = -\frac{(4 - 2x)^3}{12} \Big|_0^2 = \frac{4^3}{12} = \frac{16}{3}$$

10. A thin plate of constant density 3 is bounded by the lines $x = 0$, $y = x$, and the parabola $y = 2 - x^2$. What is the x -coordinate of the center of mass?

(a) $\frac{7}{16}$

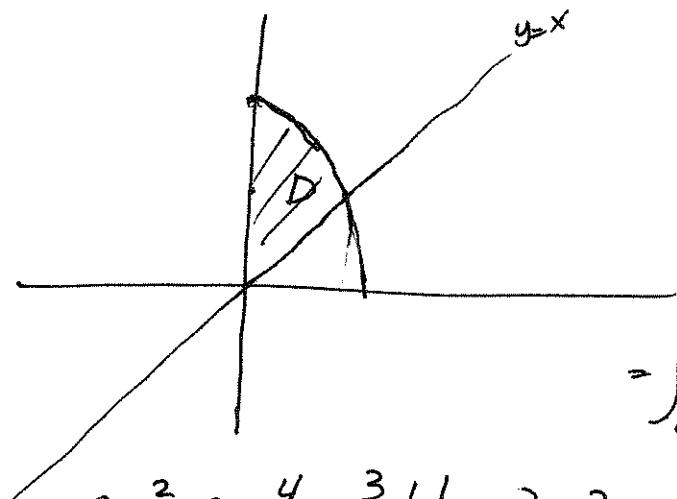
(b) $\frac{7}{6}$

(c) $\frac{2}{5}$

(d) $\frac{3}{7}$

✓ (e) $\frac{5}{14}$

(f) $\frac{5}{12}$



$$D: \begin{aligned} 0 &\leq x \leq 1 \\ x &\leq y \leq 2-x^2 \end{aligned}$$

$$\begin{aligned} \iint_D 3x \, dA &= \int_0^1 \int_x^{2-x^2} 3x \, dy \, dx \\ &= \int_0^1 3xy \Big|_{y=x}^{y=2-x^2} \, dx = \int_0^1 6x - 3x^3 - 3x^5 \, dx \end{aligned}$$

$$= 3x^2 - \frac{3}{4}x^4 - x^3 \Big|_0^1 = 3 - \frac{3}{4} - 1 = \frac{5}{4}$$

$$\begin{aligned} \iint_D 3 \, dA &= \int_0^1 \int_x^{2-x^2} 3 \, dy \, dx = \int_0^1 3y \Big|_x^{2-x^2} \, dx = \int_0^1 6 - 3x^2 - 3x \, dx = 6x - x^3 - \frac{3}{2}x^2 \Big|_0^1 \\ &= 6 - 1 - 3 - \frac{7}{2} \end{aligned}$$

11. What is the work done by the force $\mathbf{F} = \langle xy, y - x \rangle$ in moving a particle along the line segment from $(2, 3)$ to $(1, 1)$?

(a) $\frac{29}{6}$

(b) $\frac{17}{6}$

\int (c) $-\frac{25}{6}$

(d) $\frac{15}{6}$

(e) $-\frac{15}{6}$

(f) $-\frac{29}{6}$

$$C: \langle 2-t, 3-2t \rangle \quad 0 \leq t \leq 1$$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \langle (2-t)(3-2t), 1-t \rangle \cdot \langle -1, -2 \rangle dt$$

$$= \int_0^1 -6-2t^2 + 7t-2+2t \quad dt = \int_0^1 -2t^2 + 9t - 8 \quad dt$$

$$= -\frac{2}{3}t^3 + \frac{9}{2}t^2 - 8t \Big|_0^1 = -\frac{2}{3} + \frac{9}{2} - 8 = \frac{-48 + 27 - 4}{6} = \frac{-25}{6}$$

12. What is the total mass of a wire that lies along the curve

$$C : \langle t^2 - 1, 2t \rangle, \quad 0 \leq t \leq 1$$

if the density is $\rho(t) = \frac{3t}{2}$.

(a) $2\sqrt{2}$

(b) $4\sqrt{2} - 2$

(c) $2\sqrt{2} - 1$

(d) $4\sqrt{2}$

(e) $4\sqrt{2} + 2$

(f) $2\sqrt{2} + 2$

$$\begin{aligned} \int_0^1 \frac{3t}{2} \sqrt{(2t)^2 + 4} dt &= \int_0^1 3t \sqrt{t^2 + 1} dt = \left(t^2 + 1 \right)^{\frac{3}{2}} \Big|_0^1 \\ &= \sqrt{8} - 1 \end{aligned}$$

13. Let C be the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ positively oriented. If $\mathbf{F} = \langle xy, 1 \rangle$, then evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

(a) $12\pi - 2$

(b) $6\pi + 2$

(c) 6π

(d) 4π

(e) 12π

\checkmark (f) 0

$C: \langle 2\cos t, 3\sin t \rangle \quad 0 \leq t \leq 2\pi$

$$\begin{aligned} \int_C \mathbf{F} \cdot d\mathbf{r} &= \int_0^{2\pi} \langle 6\cos t \sin t, 1 \rangle \cdot \langle -2\sin t, 3\cos t \rangle dt \\ &= \int_0^{2\pi} -12\sin^2 t \cos t + 3\cos^2 t dt = -4\sin^3 t + 3\sin t \Big|_0^{2\pi} = 0 \end{aligned}$$

14. Which of the following vector fields are conservative?

$$\mathbf{F}_1 = \langle 3x^2 + 2xy, x^2 + 1 \rangle$$

$$\mathbf{F}_2 = \langle \sin(xy), x \sin y \rangle$$

$$\mathbf{F}_3 = \langle xe^y, e^{xy} \rangle$$

✓(a) Only F_1 .

(b) Only F_2 .

(c) Only F_3

(d) None of them is conservative.

(e) Only F_1 and F_3 are conservative.

(f) Only F_2 and F_3 are conservative.

15. Evaluate

$$\int_C 2x \cos y \, dx - x^2 \sin y \, dy$$

if C is the parabola $y = (x - 1)^2$ from $(1, 0)$ to $(0, 1)$.

(a) 0

(b) $\cos 1$

(c) 1

✓ (d) -1

(e) $\frac{\cos 1}{2}$

(f) $\sqrt{2}$

$\langle 2x \cos y, -x^2 \sin y \rangle$ is conservative.

If $f(x, y) = x^2 \cos y$, $f_x = 2x \cos y$, $f_y = -x^2 \sin y$

so

$$\begin{aligned} \int_C 2x \cos y \, dx - x^2 \sin y \, dy &= f(0, 1) - f(1, 0) \\ &= 0 - 1 = -1 \end{aligned}$$

16. Evaluate the line integral

$$\int_C x^2 dx + (y^3 + 2yx) dy$$

where C is the triangle cut out with lines $x = 1$, $x + y = 2$, and $y = x - 2$ oriented counter clockwise.

\int (a) 0

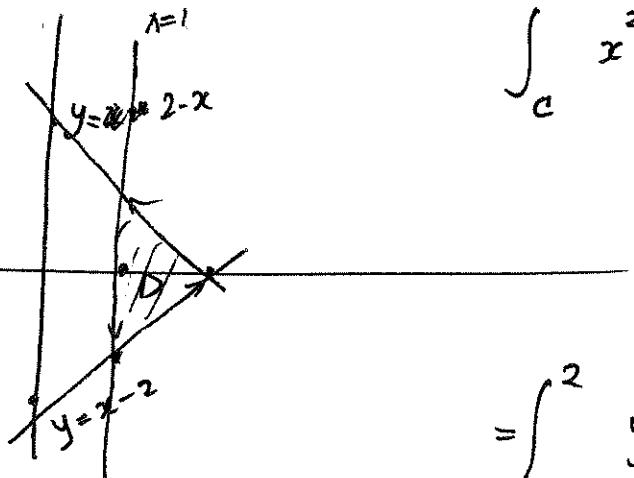
(b) 2

(c) $\frac{6}{5}$

(d) $\frac{8}{3}$

(e) -2

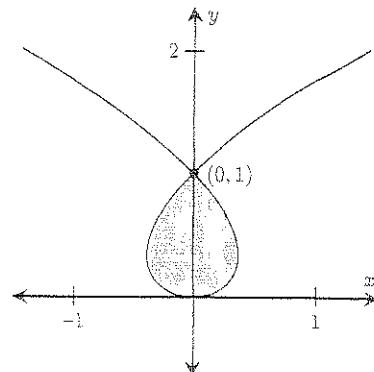
(f) -4



Green's theorem

$$\begin{aligned}
 \int_C x^2 dx + (y^3 + 2yx) dy &= \iint_D 2y dA \\
 &= \int_1^2 \int_{x-2}^{2-x} 2y dy dx \\
 &= \int_1^2 y^2 \Big|_{y=x-2}^{y=2-x} dx = \int_1^2 ((2-x)^2 - (x-2)^2) dx \\
 &= \int_1^2 0 dx = 0
 \end{aligned}$$

17. The following picture shows the curve parametrized by $\langle t - t^3, t^2 \rangle$. Use Green's Theorem to find the area of the shaded region.



(a) $\frac{6}{15}$

If $\langle t - t^3, t^2 \rangle = (0,1)$, then $t = \pm 1$

(b) $\frac{12}{15}$

$$\text{area} = \frac{1}{2} \int_C -y \, dx + x \, dy$$

(c) $\frac{3}{15}$

$$= \frac{1}{2} \int_{-1}^1 -t^2(1-3t^2) + (t-t^3)(2t) \, dt$$

(d) $\frac{4}{15}$

$$= \frac{1}{2} \int_{-1}^1 -t^2 + 3t^4 + 2t^2 - 2t^4 \, dt$$

(e) $\frac{16}{15}$

$$= \frac{1}{2} \int_{-1}^1 t^2 + t^4 \, dt = \frac{1}{2} \left(\frac{t^3}{3} + \frac{t^5}{5} \right) \Big|_{-1}^1$$

\checkmark (f) $\frac{8}{15}$

$$= \frac{8}{15}$$