

19. $\mathbf{r}(t) = (\sin t)\mathbf{i} + (t^2 - \cos t)\mathbf{j} + e^t\mathbf{k} \Rightarrow \mathbf{v}(t) = (\cos t)\mathbf{i} + (2t + \sin t)\mathbf{j} + e^t\mathbf{k}; t_0 = 0 \Rightarrow \mathbf{v}(t_0) = \mathbf{i} + \mathbf{k}$ and
 $\mathbf{r}(t_0) = \mathbf{P}_0 = (0, -1, 1) \Rightarrow x = 0 + t = t, y = -1,$ and $z = 1 + t$ are parametric equations of the tangent line
20. $\mathbf{r}(t) = (2 \sin t)\mathbf{i} + (2 \cos t)\mathbf{j} + 5t\mathbf{k} \Rightarrow \mathbf{v}(t) = (2 \cos t)\mathbf{i} - (2 \sin t)\mathbf{j} + 5\mathbf{k}; t_0 = 4\pi \Rightarrow \mathbf{v}(t_0) = 2\mathbf{i} + 5\mathbf{k}$ and
 $\mathbf{r}(t_0) = \mathbf{P}_0 = (0, 2, 20\pi) \Rightarrow x = 0 + 2t = 2t, y = 2,$ and $z = 20\pi + 5t$ are parametric equations of the tangent line
21. $\mathbf{r}(t) = (a \sin t)\mathbf{i} + (a \cos t)\mathbf{j} + btk \Rightarrow \mathbf{v}(t) = (a \cos t)\mathbf{i} - (a \sin t)\mathbf{j} + b\mathbf{k}; t_0 = 2\pi \Rightarrow \mathbf{v}(t_0) = a\mathbf{i} + b\mathbf{k}$ and
 $\mathbf{r}(t_0) = \mathbf{P}_0 = (0, a, 2b\pi) \Rightarrow x = 0 + at = at, y = a,$ and $z = 2\pi b + bt$ are parametric equations of the tangent line
22. $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + (\sin 2t)\mathbf{k} \Rightarrow \mathbf{v}(t) = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j} + (2 \cos 2t)\mathbf{k}; t_0 = \frac{\pi}{2} \Rightarrow \mathbf{v}(t_0) = -\mathbf{i} - 2\mathbf{k}$ and
 $\mathbf{r}(t_0) = \mathbf{P}_0 = (0, 1, 0) \Rightarrow x = 0 - t = -t, y = 1,$ and $z = 0 - 2t = -2t$ are parametric equations of the tangent line

23. (a) $\mathbf{v}(t) = -(\sin t)\mathbf{i} + (\cos t)\mathbf{j} \Rightarrow \mathbf{a}(t) = -(\cos t)\mathbf{i} - (\sin t)\mathbf{j};$
 (i) $|\mathbf{v}(t)| = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1 \Rightarrow$ constant speed;
 (ii) $\mathbf{v} \cdot \mathbf{a} = (\sin t)(\cos t) - (\cos t)(\sin t) = 0 \Rightarrow$ yes, orthogonal;
 (iii) counterclockwise movement;
 (iv) yes, $\mathbf{r}(0) = \mathbf{i} + 0\mathbf{j}$
- (b) $\mathbf{v}(t) = -(2 \sin 2t)\mathbf{i} + (2 \cos 2t)\mathbf{j} \Rightarrow \mathbf{a}(t) = -(4 \cos 2t)\mathbf{i} - (4 \sin 2t)\mathbf{j};$
 (i) $|\mathbf{v}(t)| = \sqrt{4 \sin^2 2t + 4 \cos^2 2t} = 2 \Rightarrow$ constant speed;
 (ii) $\mathbf{v} \cdot \mathbf{a} = 8 \sin 2t \cos 2t - 8 \cos 2t \sin 2t = 0 \Rightarrow$ yes, orthogonal;
 (iii) counterclockwise movement;
 (iv) yes, $\mathbf{r}(0) = \mathbf{i} + 0\mathbf{j}$
- (c) $\mathbf{v}(t) = -\sin(t - \frac{\pi}{2})\mathbf{i} + \cos(t - \frac{\pi}{2})\mathbf{j} \Rightarrow \mathbf{a}(t) = -\cos(t - \frac{\pi}{2})\mathbf{i} - \sin(t - \frac{\pi}{2})\mathbf{j};$
 (i) $|\mathbf{v}(t)| = \sqrt{\sin^2(t - \frac{\pi}{2}) + \cos^2(t - \frac{\pi}{2})} = 1 \Rightarrow$ constant speed;
 (ii) $\mathbf{v} \cdot \mathbf{a} = \sin(t - \frac{\pi}{2})\cos(t - \frac{\pi}{2}) - \cos(t - \frac{\pi}{2})\sin(t - \frac{\pi}{2}) = 0 \Rightarrow$ yes, orthogonal;
 (iii) counterclockwise movement;
 (iv) no, $\mathbf{r}(0) = 0\mathbf{i} - \mathbf{j}$ instead of $\mathbf{i} + 0\mathbf{j}$
- (d) $\mathbf{v}(t) = -(\sin t)\mathbf{i} - (\cos t)\mathbf{j} \Rightarrow \mathbf{a}(t) = -(\cos t)\mathbf{i} + (\sin t)\mathbf{j};$
 (i) $|\mathbf{v}(t)| = \sqrt{(-\sin t)^2 + (-\cos t)^2} = 1 \Rightarrow$ constant speed;
 (ii) $\mathbf{v} \cdot \mathbf{a} = (\sin t)(\cos t) - (\cos t)(\sin t) = 0 \Rightarrow$ yes, orthogonal;
 (iii) clockwise movement;
 (iv) yes, $\mathbf{r}(0) = \mathbf{i} - 0\mathbf{j}$
- (e) $\mathbf{v}(t) = -(2t \sin t)\mathbf{i} + (2t \cos t)\mathbf{j} \Rightarrow \mathbf{a}(t) = -(2 \sin t + 2t \cos t)\mathbf{i} + (2 \cos t - 2t \sin t)\mathbf{j};$
 (i) $|\mathbf{v}(t)| = \sqrt{[-(2t \sin t)]^2 + (2t \cos t)^2} = \sqrt{4t^2(\sin^2 t + \cos^2 t)} = 2|t| = 2t, t \geq 0$
 \Rightarrow variable speed;
 (ii) $\mathbf{v} \cdot \mathbf{a} = 4(t \sin^2 t + t^2 \sin t \cos t) + 4(t \cos^2 t - t^2 \cos t \sin t) = 4t \neq 0$ in general
 \Rightarrow not orthogonal in general;
 (iii) counterclockwise movement;
 (iv) yes, $\mathbf{r}(0) = \mathbf{i} + 0\mathbf{j}$

24. Let $\mathbf{p} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ denote the position vector of the point $(2, 2, 1)$ and let, $\mathbf{u} = \frac{1}{\sqrt{2}}\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j}$ and $\mathbf{v} = \frac{1}{\sqrt{3}}\mathbf{i} + \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}$.
 Then $\mathbf{r}(t) = \mathbf{p} + (\cos t)\mathbf{u} + (\sin t)\mathbf{v}$. Note that $(2, 2, 1)$ is a point on the plane and $\mathbf{n} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$ is normal to the plane. Moreover, \mathbf{u} and \mathbf{v} are orthogonal unit vectors with $\mathbf{u} \cdot \mathbf{n} = \mathbf{v} \cdot \mathbf{n} = 0 \Rightarrow \mathbf{u}$ and \mathbf{v} are parallel to the plane. Therefore, $\mathbf{r}(t)$ identifies a point that lies in the plane for each t . Also, for each t , $(\cos t)\mathbf{u} + (\sin t)\mathbf{v}$ is a unit vector. Starting at the point $(2 + \frac{1}{\sqrt{2}}, 2 - \frac{1}{\sqrt{2}}, 1)$ the vector $\mathbf{r}(t)$ traces out a circle of radius 1 and center $(2, 2, 1)$ in the plane $x + y - 2z = 2$.

31. (a) (Assuming that "x" is zero at the point of impact:)

$$\mathbf{r}(t) = (x(t))\mathbf{i} + (y(t))\mathbf{j}; \text{ where } x(t) = (35 \cos 27^\circ)t \text{ and } y(t) = 4 + (35 \sin 27^\circ)t - 16t^2.$$

$$(b) y_{\max} = \frac{(v_0 \sin \alpha)^2}{2g} + 4 = \frac{(35 \sin 27^\circ)^2}{64} + 4 \approx 7.945 \text{ feet, which is reached at } t = \frac{v_0 \sin \alpha}{g} = \frac{35 \sin 27^\circ}{32} \approx 0.497 \text{ seconds.}$$

- (c) For the time, solve
- $y = 4 + (35 \sin 27^\circ)t - 16t^2 = 0$
- for
- t
- , using the quadratic formula

$$t = \frac{35 \sin 27^\circ + \sqrt{(-35 \sin 27^\circ)^2 + 256}}{32} \approx 1.201 \text{ sec. Then the range is about } x(1.201) = (35 \cos 27^\circ)(1.201) \approx 37.453 \text{ feet.}$$

- (d) For the time, solve
- $y = 4 + (35 \sin 27^\circ)t - 16t^2 = 7$
- for
- t
- , using the quadratic formula

$$t = \frac{35 \sin 27^\circ + \sqrt{(-35 \sin 27^\circ)^2 - 192}}{32} \approx 0.254 \text{ and } 0.740 \text{ seconds. At those times the ball is about } x(0.254) = (35 \cos 27^\circ)(0.254) \approx 7.921 \text{ feet and } x(0.740) = (35 \cos 27^\circ)(0.740) \approx 23.077 \text{ feet the impact point, or about } 37.453 - 7.921 \approx 29.532 \text{ feet and } 37.453 - 23.077 \approx 14.376 \text{ feet from the landing spot.}$$

- (e) Yes. It changes things because the ball won't clear the net (
- $y_{\max} \approx 7.945$
-).

32. $x = x_0 + (v_0 \cos \alpha)t = 0 + (v_0 \cos 40^\circ)t \approx 0.766 v_0 t$ and $y = y_0 + (v_0 \sin \alpha)t - \frac{1}{2}gt^2 = 6.5 + (v_0 \sin 40^\circ)t - 16t^2 \approx 6.5 + 0.643 v_0 t - 16t^2$; now the shot went 73.833 ft $\Rightarrow 73.833 = 0.766 v_0 t \Rightarrow t \approx \frac{96.383}{v_0}$ sec; the shot lands when $y = 0 \Rightarrow 0 = 6.5 + (0.643)(96.383) - 16 \left(\frac{96.383}{v_0}\right)^2 \Rightarrow 0 \approx 68.474 - \frac{148.635}{v_0^2} \Rightarrow v_0 \approx \sqrt{\frac{148.635}{68.474}} \approx 46.6$ ft/sec, the shot's initial speed

11.3 ARC LENGTH IN SPACE

- $\mathbf{r} = (2 \cos t)\mathbf{i} + (2 \sin t)\mathbf{j} + \sqrt{5}t\mathbf{k} \Rightarrow \mathbf{v} = (-2 \sin t)\mathbf{i} + (2 \cos t)\mathbf{j} + \sqrt{5}\mathbf{k}$
 $\Rightarrow |\mathbf{v}| = \sqrt{(-2 \sin t)^2 + (2 \cos t)^2 + (\sqrt{5})^2} = \sqrt{4 \sin^2 t + 4 \cos^2 t + 5} = 3; \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$
 $= \left(-\frac{2}{3} \sin t\right)\mathbf{i} + \left(\frac{2}{3} \cos t\right)\mathbf{j} + \frac{\sqrt{5}}{3}\mathbf{k}$ and Length $= \int_0^\pi |\mathbf{v}| dt = \int_0^\pi 3 dt = [3t]_0^\pi = 3\pi$
- $\mathbf{r} = (6 \sin 2t)\mathbf{i} + (6 \cos 2t)\mathbf{j} + 5t\mathbf{k} \Rightarrow \mathbf{v} = (12 \cos 2t)\mathbf{i} + (-12 \sin 2t)\mathbf{j} + 5\mathbf{k}$
 $\Rightarrow |\mathbf{v}| = \sqrt{(12 \cos 2t)^2 + (-12 \sin 2t)^2 + 5^2} = \sqrt{144 \cos^2 2t + 144 \sin^2 2t + 25} = 13; \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$
 $= \left(\frac{12}{13} \cos 2t\right)\mathbf{i} - \left(\frac{12}{13} \sin 2t\right)\mathbf{j} + \frac{5}{13}\mathbf{k}$ and Length $= \int_0^\pi |\mathbf{v}| dt = \int_0^\pi 13 dt = [13t]_0^\pi = 13\pi$
- $\mathbf{r} = t\mathbf{i} + \frac{2}{3}t^{3/2}\mathbf{k} \Rightarrow \mathbf{v} = \mathbf{i} + t^{1/2}\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{1^2 + (t^{1/2})^2} = \sqrt{1+t}; \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{1+t}}\mathbf{i} + \frac{\sqrt{t}}{\sqrt{1+t}}\mathbf{k}$
 and Length $= \int_0^8 \sqrt{1+t} dt = \left[\frac{2}{3}(1+t)^{3/2}\right]_0^8 = \frac{52}{3}$
- $\mathbf{r} = (2+t)\mathbf{i} - (t+1)\mathbf{j} + t\mathbf{k} \Rightarrow \mathbf{v} = \mathbf{i} - \mathbf{j} + \mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}; \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{1}{\sqrt{3}}\mathbf{i} - \frac{1}{\sqrt{3}}\mathbf{j} + \frac{1}{\sqrt{3}}\mathbf{k}$
 and Length $= \int_0^3 \sqrt{3} dt = \left[\sqrt{3}t\right]_0^3 = 3\sqrt{3}$
- $\mathbf{r} = (\cos^3 t)\mathbf{j} + (\sin^3 t)\mathbf{k} \Rightarrow \mathbf{v} = (-3 \cos^2 t \sin t)\mathbf{j} + (3 \sin^2 t \cos t)\mathbf{k} \Rightarrow |\mathbf{v}|$
 $= \sqrt{(-3 \cos^2 t \sin t)^2 + (3 \sin^2 t \cos t)^2} = \sqrt{(9 \cos^2 t \sin^2 t)(\cos^2 t + \sin^2 t)} = 3 |\cos t \sin t|;$
 $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{-3 \cos^2 t \sin t}{3 |\cos t \sin t|}\mathbf{j} + \frac{3 \sin^2 t \cos t}{3 |\cos t \sin t|}\mathbf{k} = (-\cos t)\mathbf{j} + (\sin t)\mathbf{k}, \text{ if } 0 \leq t \leq \frac{\pi}{2}, \text{ and}$
 Length $= \int_0^{\pi/2} 3 |\cos t \sin t| dt = \int_0^{\pi/2} 3 \cos t \sin t dt = \int_0^{\pi/2} \frac{3}{2} \sin 2t dt = \left[-\frac{3}{4} \cos 2t\right]_0^{\pi/2} = \frac{3}{2}$
- $\mathbf{r} = 6t^3\mathbf{i} - 2t^3\mathbf{j} - 3t^3\mathbf{k} \Rightarrow \mathbf{v} = 18t^2\mathbf{i} - 6t^2\mathbf{j} - 9t^2\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{(18t^2)^2 + (-6t^2)^2 + (-9t^2)^2} = \sqrt{441t^4} = 21t^2;$
 $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{18t^2}{21t^2}\mathbf{i} - \frac{6t^2}{21t^2}\mathbf{j} - \frac{9t^2}{21t^2}\mathbf{k} = \frac{6}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$ and Length $= \int_1^2 21t^2 dt = [7t^3]_1^2 = 49$

7. $\mathbf{r} = (t \cos t)\mathbf{i} + (t \sin t)\mathbf{j} + \frac{2\sqrt{2}}{3}t^{3/2}\mathbf{k} \Rightarrow \mathbf{v} = (\cos t - t \sin t)\mathbf{i} + (\sin t + t \cos t)\mathbf{j} + (\sqrt{2}t^{1/2})\mathbf{k}$
 $\Rightarrow |\mathbf{v}| = \sqrt{(\cos t - t \sin t)^2 + (\sin t + t \cos t)^2 + (\sqrt{2}t)^2} = \sqrt{1 + t^2 + 2t} = \sqrt{(t+1)^2} = |t+1| = t+1, \text{ if } t \geq 0;$
 $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{\cos t - t \sin t}{t+1}\right)\mathbf{i} + \left(\frac{\sin t + t \cos t}{t+1}\right)\mathbf{j} + \left(\frac{\sqrt{2}t^{1/2}}{t+1}\right)\mathbf{k}$ and Length $= \int_0^\pi (t+1) dt = \left[\frac{t^2}{2} + t\right]_0^\pi = \frac{\pi^2}{2} + \pi$
8. $\mathbf{r} = (t \sin t + \cos t)\mathbf{i} + (t \cos t - \sin t)\mathbf{j} \Rightarrow \mathbf{v} = (\sin t + t \cos t - \sin t)\mathbf{i} + (\cos t - t \sin t - \cos t)\mathbf{j}$
 $= (t \cos t)\mathbf{i} - (t \sin t)\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{(t \cos t)^2 + (-t \sin t)^2} = \sqrt{t^2} = |t| = t \text{ if } \sqrt{2} \leq t \leq 2; \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$
 $= \left(\frac{t \cos t}{t}\right)\mathbf{i} - \left(\frac{t \sin t}{t}\right)\mathbf{j} = (\cos t)\mathbf{i} - (\sin t)\mathbf{j}$ and Length $= \int_{\sqrt{2}}^2 t dt = \left[\frac{t^2}{2}\right]_{\sqrt{2}}^2 = 1$
9. Let $P(t_0)$ denote the point. Then $\mathbf{v} = (5 \cos t)\mathbf{i} - (5 \sin t)\mathbf{j} + 12\mathbf{k}$ and $26\pi = \int_0^{t_0} \sqrt{25 \cos^2 t + 25 \sin^2 t + 144} dt$
 $= \int_0^{t_0} 13 dt = 13t_0 \Rightarrow t_0 = 2\pi$, and the point is $P(2\pi) = (5 \sin 2\pi, 5 \cos 2\pi, 24\pi) = (0, 5, 24\pi)$
10. Let $P(t_0)$ denote the point. Then $\mathbf{v} = (12 \cos t)\mathbf{i} + (12 \sin t)\mathbf{j} + 5\mathbf{k}$ and
 $-13\pi = \int_0^{t_0} \sqrt{144 \cos^2 t + 144 \sin^2 t + 25} dt = \int_0^{t_0} 13 dt = 13t_0 \Rightarrow t_0 = -\pi$, and the point is
 $P(-\pi) = (12 \sin(-\pi), -12 \cos(-\pi), -5\pi) = (0, 12, -5\pi)$
11. $\mathbf{r} = (4 \cos t)\mathbf{i} + (4 \sin t)\mathbf{j} + 3t\mathbf{k} \Rightarrow \mathbf{v} = (-4 \sin t)\mathbf{i} + (4 \cos t)\mathbf{j} + 3\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{(-4 \sin t)^2 + (4 \cos t)^2 + 3^2}$
 $= \sqrt{25} = 5 \Rightarrow s(t) = \int_0^t 5 d\tau = 5t \Rightarrow \text{Length} = s\left(\frac{\pi}{2}\right) = \frac{5\pi}{2}$
12. $\mathbf{r} = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j} \Rightarrow \mathbf{v} = (-\sin t + \sin t + t \cos t)\mathbf{i} + (\cos t - \cos t + t \sin t)\mathbf{j}$
 $= (t \cos t)\mathbf{i} + (t \sin t)\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{(t \cos t)^2 + (t \sin t)^2} = \sqrt{t^2} = t, \text{ since } \frac{\pi}{2} \leq t \leq \pi \Rightarrow s(t) = \int_0^t \tau d\tau = \frac{t^2}{2}$
 $\Rightarrow \text{Length} = s(\pi) - s\left(\frac{\pi}{2}\right) = \frac{\pi^2}{2} - \frac{(\frac{\pi}{2})^2}{2} = \frac{3\pi^2}{8}$
13. $\mathbf{r} = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j} + e^t\mathbf{k} \Rightarrow \mathbf{v} = (e^t \cos t - e^t \sin t)\mathbf{i} + (e^t \sin t + e^t \cos t)\mathbf{j} + e^t\mathbf{k}$
 $\Rightarrow |\mathbf{v}| = \sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2 + (e^t)^2} = \sqrt{3e^{2t}} = \sqrt{3}e^t \Rightarrow s(t) = \int_0^t \sqrt{3}e^\tau d\tau$
 $= \sqrt{3}e^t - \sqrt{3} \Rightarrow \text{Length} = s(0) - s(-\ln 4) = 0 - (\sqrt{3}e^{-\ln 4} - \sqrt{3}) = \frac{3\sqrt{3}}{4}$
14. $\mathbf{r} = (1 + 2t)\mathbf{i} + (1 + 3t)\mathbf{j} + (6 - 6t)\mathbf{k} \Rightarrow \mathbf{v} = 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{2^2 + 3^2 + (-6)^2} = 7 \Rightarrow s(t) = \int_0^t 7 d\tau = 7t$
 $\Rightarrow \text{Length} = s(0) - s(-1) = 0 - (-7) = 7$
15. $\mathbf{r} = (\sqrt{2t})\mathbf{i} + (\sqrt{2t})\mathbf{j} + (1 - t^2)\mathbf{k} \Rightarrow \mathbf{v} = \sqrt{2}\mathbf{i} + \sqrt{2}\mathbf{j} - 2t\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2 + (-2t)^2} = \sqrt{4 + 4t^2}$
 $= 2\sqrt{1 + t^2} \Rightarrow \text{Length} = \int_0^1 2\sqrt{1 + t^2} dt = \left[2\left(\frac{1}{2}\sqrt{1 + t^2} + \frac{1}{2}\ln(t + \sqrt{1 + t^2})\right)\right]_0^1 = \sqrt{2} + \ln(1 + \sqrt{2})$

$$4. \mathbf{r} = (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j} \Rightarrow \mathbf{v} = (t \cos t)\mathbf{i} + (t \sin t)\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{(t \cos t)^2 + (t \sin t)^2} = \sqrt{t^2} = |t| = t, \text{ since } t > 0 \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{(t \cos t)\mathbf{i} + (t \sin t)\mathbf{j}}{t} = (\cos t)\mathbf{i} + (\sin t)\mathbf{j}; \frac{d\mathbf{T}}{dt} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j} \\ \Rightarrow \left| \frac{d\mathbf{T}}{dt} \right| = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1 \Rightarrow \mathbf{N} = \frac{d\mathbf{T}}{dt} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j}; \kappa = \frac{1}{|\mathbf{v}|} \cdot \left| \frac{d\mathbf{T}}{dt} \right| = \frac{1}{t} \cdot 1 = \frac{1}{t}$$

$$5. (a) \kappa(x) = \frac{1}{|\mathbf{v}(x)|} \cdot \left| \frac{d\mathbf{T}(x)}{dx} \right|. \text{ Now, } \mathbf{v} = \mathbf{i} + f'(x)\mathbf{j} \Rightarrow |\mathbf{v}(x)| = \sqrt{1 + [f'(x)]^2} \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} \\ = (1 + [f'(x)]^2)^{-1/2} \mathbf{i} + f'(x)(1 + [f'(x)]^2)^{-1/2} \mathbf{j}. \text{ Thus } \frac{d\mathbf{T}}{dx}(x) = \frac{-f'(x)f''(x)}{(1 + [f'(x)]^2)^{3/2}} \mathbf{i} + \frac{f''(x)}{(1 + [f'(x)]^2)^{3/2}} \mathbf{j}$$

$$\Rightarrow \left| \frac{d\mathbf{T}(x)}{dx} \right| = \sqrt{\left[\frac{-f'(x)f''(x)}{(1 + [f'(x)]^2)^{3/2}} \right]^2 + \left[\frac{f''(x)}{(1 + [f'(x)]^2)^{3/2}} \right]^2} = \sqrt{\frac{[f''(x)]^2(1 + [f'(x)]^2)}{(1 + [f'(x)]^2)^3}} = \frac{|f''(x)|}{1 + [f'(x)]^2}$$

$$\text{Thus } \kappa(x) = \frac{1}{(1 + [f'(x)]^2)^{1/2}} \cdot \frac{|f''(x)|}{1 + [f'(x)]^2} = \frac{|f''(x)|}{(1 + [f'(x)]^2)^{3/2}}$$

$$(b) y = \ln(\cos x) \Rightarrow \frac{dy}{dx} = \left(\frac{1}{\cos x}\right)(-\sin x) = -\tan x \Rightarrow \frac{d^2y}{dx^2} = -\sec^2 x \Rightarrow \kappa = \frac{|-\sec^2 x|}{[1 + (-\tan x)^2]^{3/2}} = \frac{\sec^2 x}{|\sec^2 x|} \\ = \frac{1}{\sec x} = \cos x, \text{ since } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(c) Note that $f''(x) = 0$ at an inflection point.

$$6. (a) \mathbf{r} = f(t)\mathbf{i} + g(t)\mathbf{j} = x\mathbf{i} + y\mathbf{j} \Rightarrow \mathbf{v} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{\dot{x}^2 + \dot{y}^2} \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \mathbf{i} + \frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} \mathbf{j}$$

$$\frac{d\mathbf{T}}{dt} = \frac{\dot{y}(\dot{y}\dot{x} - \dot{x}\dot{y})}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \mathbf{i} + \frac{\dot{x}(\dot{x}\dot{y} - \dot{y}\dot{x})}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \mathbf{j} \Rightarrow \left| \frac{d\mathbf{T}}{dt} \right| = \sqrt{\left[\frac{\dot{y}(\dot{y}\dot{x} - \dot{x}\dot{y})}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \right]^2 + \left[\frac{\dot{x}(\dot{x}\dot{y} - \dot{y}\dot{x})}{(\dot{x}^2 + \dot{y}^2)^{3/2}} \right]^2} = \sqrt{\frac{(y^2 + \dot{x}^2)(\dot{y}\dot{x} - \dot{x}\dot{y})^2}{(\dot{x}^2 + \dot{y}^2)^3}} \\ = \frac{|\dot{y}\dot{x} - \dot{x}\dot{y}|}{|\dot{x}^2 + \dot{y}^2|}; \kappa = \frac{1}{|\mathbf{v}|} \cdot \left| \frac{d\mathbf{T}}{dt} \right| = \frac{1}{\sqrt{\dot{x}^2 + \dot{y}^2}} \cdot \frac{|\dot{y}\dot{x} - \dot{x}\dot{y}|}{|\dot{x}^2 + \dot{y}^2|} = \frac{|\dot{x}\dot{y} - \dot{y}\dot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$$

$$(b) \mathbf{r}(t) = t\mathbf{i} + \ln(\sin t)\mathbf{j}, 0 < t < \pi \Rightarrow x = t \text{ and } y = \ln(\sin t) \Rightarrow \dot{x} = 1, \ddot{x} = 0; \dot{y} = \frac{\cos t}{\sin t} = \cot t, \ddot{y} = -\csc^2 t \\ \Rightarrow \kappa = \frac{|-\csc^2 t - 0|}{(1 + \cot^2 t)^{3/2}} = \frac{\csc^2 t}{\csc^3 t} = \sin t$$

$$(c) \mathbf{r}(t) = \tan^{-1}(\sinh t)\mathbf{i} + \ln(\cosh t)\mathbf{j} \Rightarrow x = \tan^{-1}(\sinh t) \text{ and } y = \ln(\cosh t) \Rightarrow \dot{x} = \frac{\cosh t}{1 + \sinh^2 t} = \frac{1}{\cosh t} \\ = \operatorname{sech} t, \ddot{x} = -\operatorname{sech} t \tanh t; \dot{y} = \frac{\sinh t}{\cosh t} = \tanh t, \ddot{y} = \operatorname{sech}^2 t \Rightarrow \kappa = \frac{|\operatorname{sech}^3 t + \operatorname{sech} t \tanh^2 t|}{(\operatorname{sech}^2 t + \tanh^2 t)^{3/2}} = |\operatorname{sech} t| = \operatorname{sech} t$$

7. (a) $\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} \Rightarrow \mathbf{v} = f'(t)\mathbf{i} + g'(t)\mathbf{j}$ is tangent to the curve at the point $(f(t), g(t))$;

$\mathbf{n} \cdot \mathbf{v} = [-g'(t)\mathbf{i} + f'(t)\mathbf{j}] \cdot [f'(t)\mathbf{i} + g'(t)\mathbf{j}] = -g'(t)f'(t) + f'(t)g'(t) = 0$; $-\mathbf{n} \cdot \mathbf{v} = -(\mathbf{n} \cdot \mathbf{v}) = 0$; thus, \mathbf{n} and $-\mathbf{n}$ are both normal to the curve at the point

(b) $\mathbf{r}(t) = t\mathbf{i} + e^{2t}\mathbf{j} \Rightarrow \mathbf{v} = \mathbf{i} + 2e^{2t}\mathbf{j} \Rightarrow \mathbf{n} = -2e^{2t}\mathbf{i} + \mathbf{j}$ points toward the concave side of the curve; $\mathbf{N} = \frac{\mathbf{n}}{|\mathbf{n}|}$ and

$$|\mathbf{n}| = \sqrt{4e^{4t} + 1} \Rightarrow \mathbf{N} = \frac{-2e^{2t}}{\sqrt{1 + 4e^{4t}}} \mathbf{i} + \frac{1}{\sqrt{1 + 4e^{4t}}} \mathbf{j}$$

(c) $\mathbf{r}(t) = \sqrt{4 - t^2}\mathbf{i} + t\mathbf{j} \Rightarrow \mathbf{v} = \frac{-t}{\sqrt{4 - t^2}} \mathbf{i} + \mathbf{j} \Rightarrow \mathbf{n} = -\mathbf{i} - \frac{t}{\sqrt{4 - t^2}} \mathbf{j}$ points toward the concave side of the curve;

$$\mathbf{N} = \frac{\mathbf{n}}{|\mathbf{n}|} \text{ and } |\mathbf{n}| = \sqrt{1 + \frac{t^2}{4 - t^2}} = \frac{2}{\sqrt{4 - t^2}} \Rightarrow \mathbf{N} = -\frac{1}{2} (\sqrt{4 - t^2} \mathbf{i} + t\mathbf{j})$$

8. (a) $\mathbf{r}(t) = t\mathbf{i} + \frac{1}{3}t^3\mathbf{j} \Rightarrow \mathbf{v} = \mathbf{i} + t^2\mathbf{j} \Rightarrow \mathbf{n} = t^2\mathbf{i} - \mathbf{j}$ points toward the concave side of the curve when $t < 0$ and $-\mathbf{n} = -t^2\mathbf{i} + \mathbf{j}$ points toward the concave side when $t > 0 \Rightarrow \mathbf{N} = \frac{1}{\sqrt{1 + t^4}} (t^2\mathbf{i} - \mathbf{j})$ for $t < 0$ and

$$\mathbf{N} = \frac{1}{\sqrt{1 + t^4}} (-t^2\mathbf{i} + \mathbf{j}) \text{ for } t > 0$$

(b) From part (a), $|\mathbf{v}| = \sqrt{1 + t^4} \Rightarrow \mathbf{T} = \frac{1}{\sqrt{1 + t^4}} \mathbf{i} + \frac{t^2}{\sqrt{1 + t^4}} \mathbf{j} \Rightarrow \frac{d\mathbf{T}}{dt} = \frac{-2t^3}{(1 + t^4)^{3/2}} \mathbf{i} + \frac{2t}{(1 + t^4)^{3/2}} \mathbf{j} \Rightarrow \left| \frac{d\mathbf{T}}{dt} \right| = \sqrt{\frac{4t^6 + 4t^2}{(1 + t^4)^3}}$

$$= \frac{2|t|}{1 + t^4}; \mathbf{N} = \frac{d\mathbf{T}}{dt} = \frac{1 + t^4}{2|t|} \left(\frac{-2t^3}{(1 + t^4)^{3/2}} \mathbf{i} + \frac{2t}{(1 + t^4)^{3/2}} \mathbf{j} \right) = \frac{-t^3}{|t|\sqrt{1 + t^4}} \mathbf{i} + \frac{1}{|t|\sqrt{1 + t^4}} \mathbf{j}; t \neq 0$$

\mathbf{N} does not exist at $t = 0$, where the curve has a point of inflection; $\frac{d\mathbf{T}}{dt} \Big|_{t=0} = 0$ so the curvature $\kappa = \left| \frac{d\mathbf{T}}{ds} \right|$

$$= \left| \frac{d\mathbf{T}}{dt} \cdot \frac{dt}{ds} \right| = 0 \text{ at } t = 0 \Rightarrow \mathbf{N} = \frac{1}{\kappa} \frac{d\mathbf{T}}{ds} \text{ is undefined. Since } x = t \text{ and } y = \frac{1}{3}t^3 \Rightarrow y = \frac{1}{3}x^3, \text{ the curve is the}$$

cubic power curve which is concave down for $x = t < 0$ and concave up for $x = t > 0$.

$$\begin{aligned}
 9. \quad \mathbf{r} &= (3 \sin t)\mathbf{i} + (3 \cos t)\mathbf{j} + 4t\mathbf{k} \Rightarrow \mathbf{v} = (3 \cos t)\mathbf{i} + (-3 \sin t)\mathbf{j} + 4\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{(3 \cos t)^2 + (-3 \sin t)^2 + 4^2} \\
 &= \sqrt{25} = 5 \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{3}{5} \cos t\right)\mathbf{i} - \left(\frac{3}{5} \sin t\right)\mathbf{j} + \frac{4}{5}\mathbf{k} \Rightarrow \frac{d\mathbf{T}}{dt} = \left(-\frac{3}{5} \sin t\right)\mathbf{i} - \left(\frac{3}{5} \cos t\right)\mathbf{j} \\
 &\Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{\left(-\frac{3}{5} \sin t\right)^2 + \left(-\frac{3}{5} \cos t\right)^2} = \frac{3}{5} \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j}; \kappa = \frac{1}{5} \cdot \frac{3}{5} = \frac{3}{25}
 \end{aligned}$$

$$\begin{aligned}
 10. \quad \mathbf{r} &= (\cos t + t \sin t)\mathbf{i} + (\sin t - t \cos t)\mathbf{j} + 3t\mathbf{k} \Rightarrow \mathbf{v} = (t \cos t)\mathbf{i} + (t \sin t)\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{(t \cos t)^2 + (t \sin t)^2} = \sqrt{t^2} \\
 &= |t| = t, \text{ if } t > 0 \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = (\cos t)\mathbf{i} - (\sin t)\mathbf{j}, t > 0 \Rightarrow \frac{d\mathbf{T}}{dt} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j} \\
 &\Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1 \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j}; \kappa = \frac{1}{t} \cdot 1 = \frac{1}{t}
 \end{aligned}$$

$$\begin{aligned}
 11. \quad \mathbf{r} &= (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j} + 2t\mathbf{k} \Rightarrow \mathbf{v} = (e^t \cos t - e^t \sin t)\mathbf{i} + (e^t \sin t + e^t \cos t)\mathbf{j} \Rightarrow \\
 |\mathbf{v}| &= \sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2} = \sqrt{2e^{2t}} = e^t \sqrt{2}; \\
 \mathbf{T} &= \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\frac{\sin t + \cos t}{\sqrt{2}}\right)\mathbf{j} \Rightarrow \frac{d\mathbf{T}}{dt} = \left(\frac{-\sin t - \cos t}{\sqrt{2}}\right)\mathbf{i} + \left(\frac{\cos t - \sin t}{\sqrt{2}}\right)\mathbf{j} \\
 &\Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{\left(\frac{-\sin t - \cos t}{\sqrt{2}}\right)^2 + \left(\frac{\cos t - \sin t}{\sqrt{2}}\right)^2} = 1 \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = \left(\frac{-\cos t - \sin t}{\sqrt{2}}\right)\mathbf{i} + \left(\frac{-\sin t + \cos t}{\sqrt{2}}\right)\mathbf{j}; \\
 \kappa &= \frac{1}{|\mathbf{v}|} \cdot \left|\frac{d\mathbf{T}}{dt}\right| = \frac{1}{e^t \sqrt{2}} \cdot 1 = \frac{1}{e^t \sqrt{2}}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \mathbf{r} &= (6 \sin 2t)\mathbf{i} + (6 \cos 2t)\mathbf{j} + 5t\mathbf{k} \Rightarrow \mathbf{v} = (12 \cos 2t)\mathbf{i} - (12 \sin 2t)\mathbf{j} + 5\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{(12 \cos 2t)^2 + (-12 \sin 2t)^2 + 5^2} \\
 &= \sqrt{169} = 13 \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{12}{13} \cos 2t\right)\mathbf{i} - \left(\frac{12}{13} \sin 2t\right)\mathbf{j} + \frac{5}{13}\mathbf{k} \Rightarrow \frac{d\mathbf{T}}{dt} = \left(-\frac{24}{13} \sin 2t\right)\mathbf{i} - \left(\frac{24}{13} \cos 2t\right)\mathbf{j} \\
 &\Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{\left(-\frac{24}{13} \sin 2t\right)^2 + \left(-\frac{24}{13} \cos 2t\right)^2} = \frac{24}{13} \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = (-\sin 2t)\mathbf{i} - (\cos 2t)\mathbf{j}; \\
 \kappa &= \frac{1}{|\mathbf{v}|} \cdot \left|\frac{d\mathbf{T}}{dt}\right| = \frac{1}{13} \cdot \frac{24}{13} = \frac{24}{169}.
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \mathbf{r} &= \left(\frac{t^2}{3}\right)\mathbf{i} + \left(\frac{t^2}{2}\right)\mathbf{j}, t > 0 \Rightarrow \mathbf{v} = t^2\mathbf{i} + t\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{t^4 + t^2} = t\sqrt{t^2 + 1}, \text{ since } t > 0 \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} \\
 &= \frac{t}{\sqrt{t^2 + 1}}\mathbf{i} + \frac{1}{\sqrt{t^2 + 1}}\mathbf{j} \Rightarrow \frac{d\mathbf{T}}{dt} = \frac{1}{(t^2 + 1)^{3/2}}\mathbf{i} - \frac{t}{(t^2 + 1)^{3/2}}\mathbf{j} \Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{\left(\frac{1}{(t^2 + 1)^{3/2}}\right)^2 + \left(\frac{-t}{(t^2 + 1)^{3/2}}\right)^2} \\
 &= \sqrt{\frac{1 + t^2}{(t^2 + 1)^3}} = \frac{1}{t^2 + 1} \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = \frac{1}{\sqrt{t^2 + 1}}\mathbf{i} - \frac{t}{\sqrt{t^2 + 1}}\mathbf{j}; \kappa = \frac{1}{|\mathbf{v}|} \cdot \left|\frac{d\mathbf{T}}{dt}\right| = \frac{1}{t\sqrt{t^2 + 1}} \cdot \frac{1}{t^2 + 1} = \frac{1}{t(t^2 + 1)^{3/2}}.
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \mathbf{r} &= (\cos^3 t)\mathbf{i} + (\sin^3 t)\mathbf{j}, 0 < t < \frac{\pi}{2} \Rightarrow \mathbf{v} = (-3 \cos^2 t \sin t)\mathbf{i} + (3 \sin^2 t \cos t)\mathbf{j} \\
 &\Rightarrow |\mathbf{v}| = \sqrt{(-3 \cos^2 t \sin t)^2 + (3 \sin^2 t \cos t)^2} = \sqrt{9 \cos^4 t \sin^2 t + 9 \sin^4 t \cos^2 t} = 3 \cos t \sin t, \text{ since } 0 < t < \frac{\pi}{2} \\
 &\Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = (-\cos t)\mathbf{i} + (\sin t)\mathbf{j} \Rightarrow \frac{d\mathbf{T}}{dt} = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} \Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{\sin^2 t + \cos^2 t} = 1 \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} \\
 &= (\sin t)\mathbf{i} + (\cos t)\mathbf{j}; \kappa = \frac{1}{|\mathbf{v}|} \cdot \left|\frac{d\mathbf{T}}{dt}\right| = \frac{1}{3 \cos t \sin t} \cdot 1 = \frac{1}{3 \cos t \sin t}.
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \mathbf{r} &= t\mathbf{i} + (a \cosh \frac{t}{a})\mathbf{j}, a > 0 \Rightarrow \mathbf{v} = \mathbf{i} + (\sinh \frac{t}{a})\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{1 + \sinh^2 \left(\frac{t}{a}\right)} = \sqrt{\cosh^2 \left(\frac{t}{a}\right)} = \cosh \frac{t}{a} \\
 &\Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\operatorname{sech} \frac{t}{a}\right)\mathbf{i} + \left(\tanh \frac{t}{a}\right)\mathbf{j} \Rightarrow \frac{d\mathbf{T}}{dt} = \left(-\frac{1}{a} \operatorname{sech} \frac{t}{a} \tanh \frac{t}{a}\right)\mathbf{i} + \left(\frac{1}{a} \operatorname{sech}^2 \frac{t}{a}\right)\mathbf{j} \\
 &\Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{\frac{1}{a^2} \operatorname{sech}^2 \left(\frac{t}{a}\right) \tanh^2 \left(\frac{t}{a}\right) + \frac{1}{a^2} \operatorname{sech}^4 \left(\frac{t}{a}\right)} = \frac{1}{a} \operatorname{sech} \left(\frac{t}{a}\right) \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = \left(-\tanh \frac{t}{a}\right)\mathbf{i} + \left(\operatorname{sech} \frac{t}{a}\right)\mathbf{j}; \\
 \kappa &= \frac{1}{|\mathbf{v}|} \cdot \left|\frac{d\mathbf{T}}{dt}\right| = \frac{1}{\cosh \frac{t}{a}} \cdot \frac{1}{a} \operatorname{sech} \left(\frac{t}{a}\right) = \frac{1}{a} \operatorname{sech}^2 \left(\frac{t}{a}\right).
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \mathbf{r} &= (\cosh t)\mathbf{i} - (\sinh t)\mathbf{j} + t\mathbf{k} \Rightarrow \mathbf{v} = (\sinh t)\mathbf{i} - (\cosh t)\mathbf{j} + \mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{\sinh^2 t + (-\cosh t)^2 + 1} = \sqrt{2} \cosh t \\
 &\Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = \left(\frac{1}{\sqrt{2}} \tanh t\right)\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j} + \left(\frac{1}{\sqrt{2}} \operatorname{sech} t\right)\mathbf{k} \Rightarrow \frac{d\mathbf{T}}{dt} = \left(\frac{1}{\sqrt{2}} \operatorname{sech}^2 t\right)\mathbf{i} - \left(\frac{1}{\sqrt{2}} \operatorname{sech} t \tanh t\right)\mathbf{k} \\
 &\Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{\frac{1}{2} \operatorname{sech}^4 t + \frac{1}{2} \operatorname{sech}^2 t \tanh^2 t} = \frac{1}{\sqrt{2}} \operatorname{sech} t \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = (\operatorname{sech} t)\mathbf{i} - (\tanh t)\mathbf{k}; \\
 \kappa &= \frac{1}{|\mathbf{v}|} \cdot \left|\frac{d\mathbf{T}}{dt}\right| = \frac{1}{\sqrt{2} \cosh t} \cdot \frac{1}{\sqrt{2}} \operatorname{sech} t = \frac{1}{2} \operatorname{sech}^2 t.
 \end{aligned}$$

$$5. \mathbf{r} = t^2\mathbf{i} + \left(t + \frac{1}{3}t^3\right)\mathbf{j} + \left(t - \frac{1}{3}t^3\right)\mathbf{k} \Rightarrow \mathbf{v} = 2t\mathbf{i} + (1 + t^2)\mathbf{j} + (1 - t^2)\mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{(2t)^2 + (1 + t^2)^2 + (1 - t^2)^2}$$

$$= \sqrt{2(t^4 + 2t^2 + 1)} = \sqrt{2}(1 + t^2) \Rightarrow a_T = 2t\sqrt{2} \Rightarrow a_T(0) = 0; \mathbf{a} = 2\mathbf{i} + 2t\mathbf{j} - 2t\mathbf{k} \Rightarrow \mathbf{a}(0) = 2\mathbf{i} \Rightarrow |\mathbf{a}(0)| = 2$$

$$\Rightarrow a_N = \sqrt{|\mathbf{a}|^2 - a_T^2} = \sqrt{2^2 - 0^2} = 2 \Rightarrow \mathbf{a}(0) = (0)\mathbf{T} + 2\mathbf{N} = 2\mathbf{N}$$

$$6. \mathbf{r} = (e^t \cos t)\mathbf{i} + (e^t \sin t)\mathbf{j} + \sqrt{2}e^t\mathbf{k} \Rightarrow \mathbf{v} = (e^t \cos t - e^t \sin t)\mathbf{i} + (e^t \sin t + e^t \cos t)\mathbf{j} + \sqrt{2}e^t\mathbf{k}$$

$$\Rightarrow |\mathbf{v}| = \sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \sin t + e^t \cos t)^2 + (\sqrt{2}e^t)^2} = \sqrt{4e^{2t}} = 2e^t \Rightarrow a_T = 2e^t \Rightarrow a_T(0) = 2;$$

$$\mathbf{a} = (e^t \cos t - e^t \sin t - e^t \sin t - e^t \cos t)\mathbf{i} + (e^t \sin t + e^t \cos t + e^t \cos t - e^t \sin t)\mathbf{j} + \sqrt{2}e^t\mathbf{k}$$

$$= (-2e^t \sin t)\mathbf{i} + (2e^t \cos t)\mathbf{j} + \sqrt{2}e^t\mathbf{k} \Rightarrow \mathbf{a}(0) = 2\mathbf{j} + \sqrt{2}\mathbf{k} \Rightarrow |\mathbf{a}(0)| = \sqrt{2^2 + (\sqrt{2})^2} = \sqrt{6}$$

$$\Rightarrow a_N = \sqrt{|\mathbf{a}|^2 - a_T^2} = \sqrt{(\sqrt{6})^2 - 2^2} = \sqrt{2} \Rightarrow \mathbf{a}(0) = 2\mathbf{T} + \sqrt{2}\mathbf{N}$$

$$7. \mathbf{r} = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} - \mathbf{k} \Rightarrow \mathbf{v} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{(-\sin t)^2 + (\cos t)^2} = 1 \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

$$= (-\sin t)\mathbf{i} + (\cos t)\mathbf{j} \Rightarrow \mathbf{T}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}; \frac{d\mathbf{T}}{dt} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j} \Rightarrow \left|\frac{d\mathbf{T}}{dt}\right| = \sqrt{(-\cos t)^2 + (-\sin t)^2}$$

$$= 1 \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j} \Rightarrow \mathbf{N}\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}\mathbf{i} - \frac{\sqrt{2}}{2}\mathbf{j}; \mathbf{B} = \mathbf{T} \times \mathbf{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\sin t & \cos t & 0 \\ -\cos t & -\sin t & 0 \end{vmatrix} = \mathbf{k}$$

$$\Rightarrow \mathbf{B}\left(\frac{\pi}{4}\right) = \mathbf{k}, \text{ the normal to the osculating plane; } \mathbf{r}\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j} - \mathbf{k} \Rightarrow P = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, -1\right) \text{ lies on the}$$

$$\text{osculating plane} \Rightarrow 0\left(x - \frac{\sqrt{2}}{2}\right) + 0\left(y - \frac{\sqrt{2}}{2}\right) + (z - (-1)) = 0 \Rightarrow z = -1 \text{ is the osculating plane; } \mathbf{T} \text{ is normal}$$

$$\text{to the normal plane} \Rightarrow \left(-\frac{\sqrt{2}}{2}\right)\left(x - \frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(y - \frac{\sqrt{2}}{2}\right) + 0(z - (-1)) = 0 \Rightarrow -\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}y = 0$$

$$\Rightarrow -x + y = 0 \text{ is the normal plane; } \mathbf{N} \text{ is normal to the rectifying plane}$$

$$\Rightarrow \left(-\frac{\sqrt{2}}{2}\right)\left(x - \frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{2}}{2}\right)\left(y - \frac{\sqrt{2}}{2}\right) + 0(z - (-1)) = 0 \Rightarrow -\frac{\sqrt{2}}{2}x - \frac{\sqrt{2}}{2}y = -1 \Rightarrow x + y = \sqrt{2} \text{ is the}$$

$$\text{rectifying plane}$$

$$8. \mathbf{r} = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k} \Rightarrow \mathbf{v} = (-\sin t)\mathbf{i} + (\cos t)\mathbf{j} + \mathbf{k} \Rightarrow |\mathbf{v}| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2} \Rightarrow \mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|}$$

$$= \left(-\frac{1}{\sqrt{2}}\sin t\right)\mathbf{i} + \left(\frac{1}{\sqrt{2}}\cos t\right)\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k} \Rightarrow \frac{d\mathbf{T}}{dt} = \left(-\frac{1}{\sqrt{2}}\cos t\right)\mathbf{i} + \left(-\frac{1}{\sqrt{2}}\sin t\right)\mathbf{j} \Rightarrow \left|\frac{d\mathbf{T}}{dt}\right|$$

$$= \sqrt{\frac{1}{2}\cos^2 t + \frac{1}{2}\sin^2 t} = \frac{1}{\sqrt{2}} \Rightarrow \mathbf{N} = \frac{\left(\frac{d\mathbf{T}}{dt}\right)}{\left|\frac{d\mathbf{T}}{dt}\right|} = (-\cos t)\mathbf{i} - (\sin t)\mathbf{j}; \text{ thus } \mathbf{T}(0) = \frac{1}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k} \text{ and } \mathbf{N}(0) = -\mathbf{i}$$

$$\Rightarrow \mathbf{B}(0) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -1 & 0 & 0 \end{vmatrix} = -\frac{1}{\sqrt{2}}\mathbf{j} + \frac{1}{\sqrt{2}}\mathbf{k}, \text{ the normal to the osculating plane; } \mathbf{r}(0) = \mathbf{i} \Rightarrow P(1, 0, 0) \text{ lies on}$$

the osculating plane $\Rightarrow 0(x - 1) - \frac{1}{\sqrt{2}}(y - 0) + \frac{1}{\sqrt{2}}(z - 0) = 0 \Rightarrow y - z = 0$ is the osculating plane; \mathbf{T} is normal to the normal plane $\Rightarrow 0(x - 1) + \frac{1}{\sqrt{2}}(y - 0) + \frac{1}{\sqrt{2}}(z - 0) = 0 \Rightarrow y + z = 0$ is the normal plane; \mathbf{N} is normal to the rectifying plane $\Rightarrow -1(x - 1) + 0(y - 0) + 0(z - 0) = 0 \Rightarrow x = 1$ is the rectifying plane

$$9. \text{ By Exercise 9 in Section 11.4, } \mathbf{T} = \left(\frac{3}{5}\cos t\right)\mathbf{i} + \left(-\frac{3}{5}\sin t\right)\mathbf{j} + \frac{4}{5}\mathbf{k} \text{ and } \mathbf{N} = (-\sin t)\mathbf{i} - (\cos t)\mathbf{j} \text{ so that } \mathbf{B} = \mathbf{T} \times \mathbf{N}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{3}{5}\cos t & -\frac{3}{5}\sin t & \frac{4}{5} \\ -\sin t & -\cos t & 0 \end{vmatrix} = \left(\frac{4}{5}\cos t\right)\mathbf{i} - \left(\frac{4}{5}\sin t\right)\mathbf{j} - \frac{3}{5}\mathbf{k}. \text{ Also } \mathbf{v} = (3\cos t)\mathbf{i} + (-3\sin t)\mathbf{j} + 4\mathbf{k}$$

$$\Rightarrow \mathbf{a} = (-3\sin t)\mathbf{i} + (-3\cos t)\mathbf{j} \Rightarrow \frac{d\mathbf{a}}{dt} = (-3\cos t)\mathbf{i} + (3\sin t)\mathbf{j} \text{ and } \mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3\cos t & -3\sin t & 4 \\ -3\sin t & -3\cos t & 0 \end{vmatrix}$$

$$= (12\cos t)\mathbf{i} - (12\sin t)\mathbf{j} - 9\mathbf{k} \Rightarrow |\mathbf{v} \times \mathbf{a}|^2 = (12\cos t)^2 + (-12\sin t)^2 + (-9)^2 = 225. \text{ Thus}$$

14. By Exercise 14 in Section 13.4, $\mathbf{T} = (-\cos t)\mathbf{i} + (\sin t)\mathbf{j}$ and $\mathbf{N} = (\sin t)\mathbf{i} + (\cos t)\mathbf{j}$ so that $\mathbf{B} = \mathbf{T} \times \mathbf{N}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\cos t & \sin t & 0 \\ \sin t & \cos t & 0 \end{vmatrix} = -\mathbf{k}. \text{ Also, } \mathbf{v} = (-3 \cos^2 t \sin t)\mathbf{i} + (3 \sin^2 t \cos t)\mathbf{j}$$

$$\Rightarrow \mathbf{a} = \frac{d}{dt}(-3 \cos^2 t \sin t)\mathbf{i} + \frac{d}{dt}(3 \sin^2 t \cos t)\mathbf{j} \Rightarrow \frac{d\mathbf{a}}{dt} = \frac{d}{dt}\left(\frac{d}{dt}(-3 \cos^2 t \sin t)\right)\mathbf{i} + \frac{d}{dt}\left(\frac{d}{dt}(3 \sin^2 t \cos t)\right)\mathbf{j}$$

$$\Rightarrow \begin{vmatrix} -3 \cos^2 t \sin t & 3 \sin^2 t \cos t & 0 \\ \frac{d}{dt}(-3 \cos^2 t \sin t) & \frac{d}{dt}(3 \sin^2 t \cos t) & 0 \\ \frac{d}{dt}\left(\frac{d}{dt}(-3 \cos^2 t \sin t)\right) & \frac{d}{dt}\left(\frac{d}{dt}(3 \sin^2 t \cos t)\right) & 0 \end{vmatrix} = 0 \Rightarrow \tau = 0$$

15. By Exercise 15 in Section 11.4, $\mathbf{T} = \frac{\mathbf{v}}{|\mathbf{v}|} = (\operatorname{sech} \frac{t}{a})\mathbf{i} + (\tanh \frac{t}{a})\mathbf{j}$ and $\mathbf{N} = (-\tanh \frac{t}{a})\mathbf{i} + (\operatorname{sech} \frac{t}{a})\mathbf{j}$ so that $\mathbf{B} = \mathbf{T} \times \mathbf{N}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \operatorname{sech}(\frac{t}{a}) & \tanh(\frac{t}{a}) & 0 \\ -\tanh(\frac{t}{a}) & \operatorname{sech}(\frac{t}{a}) & 0 \end{vmatrix} = \mathbf{k}. \text{ Also, } \mathbf{v} = \mathbf{i} + (\sinh \frac{t}{a})\mathbf{j} \Rightarrow \mathbf{a} = (\frac{1}{a} \cosh \frac{t}{a})\mathbf{j} \Rightarrow \frac{d\mathbf{a}}{dt} = \frac{1}{a^2} \sinh(\frac{t}{a})\mathbf{j} \text{ so that}$$

$$\begin{vmatrix} 1 & \sinh(\frac{t}{a}) & 0 \\ 0 & \frac{1}{a} \cosh(\frac{t}{a}) & 0 \\ 0 & \frac{1}{a^2} \sinh(\frac{t}{a}) & 0 \end{vmatrix} = 0 \Rightarrow \tau = 0$$

16. By Exercise 16 in Section 11.4, $\mathbf{T} = (\frac{1}{\sqrt{2}} \tanh t)\mathbf{i} - \frac{1}{\sqrt{2}}\mathbf{j} + (\frac{1}{\sqrt{2}} \operatorname{sech} t)\mathbf{k}$ and $\mathbf{N} = (\operatorname{sech} t)\mathbf{i} - (\tanh t)\mathbf{k}$ so that

$$\mathbf{B} = \mathbf{T} \times \mathbf{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{1}{\sqrt{2}} \tanh t & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \operatorname{sech} t \\ \operatorname{sech} t & 0 & -\tanh t \end{vmatrix} = (\frac{1}{\sqrt{2}} \tanh t)\mathbf{i} + \frac{1}{\sqrt{2}}\mathbf{j} + (\frac{1}{\sqrt{2}} \operatorname{sech} t)\mathbf{k}. \text{ Also, } \mathbf{v} = (\sinh t)\mathbf{i} - (\cosh t)\mathbf{j} + \mathbf{k}$$

$$\mathbf{a} = (\cosh t)\mathbf{i} - (\sinh t)\mathbf{j} \Rightarrow \frac{d\mathbf{a}}{dt} = (\sinh t)\mathbf{i} - (\cosh t)\mathbf{j} \text{ and } \mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \sinh t & -\cosh t & 1 \\ \cosh t & -\sinh t & 0 \end{vmatrix}$$

$$= (\sinh t)\mathbf{i} + (\cosh t)\mathbf{j} + (\cosh^2 t - \sinh^2 t)\mathbf{k} = (\sinh t)\mathbf{i} + (\cosh t)\mathbf{j} + \mathbf{k} \Rightarrow |\mathbf{v} \times \mathbf{a}|^2 = \sinh^2 t + \cosh^2 t + 1. \text{ Thus}$$

$$\tau = \frac{\begin{vmatrix} \sinh t & -\cosh t & 1 \\ \cosh t & -\sinh t & 0 \\ \sinh t & -\cosh t & 0 \end{vmatrix}}{\sinh^2 t + \cosh^2 t + 1} = \frac{-1}{\sinh^2 t + \cosh^2 t + 1} = \frac{-1}{2 \cosh^2 t}.$$

17. Yes. If the car is moving along a curved path, then $\kappa \neq 0$ and $a_N = \kappa |\mathbf{v}|^2 \neq 0 \Rightarrow \mathbf{a} = a_T \mathbf{T} + a_N \mathbf{N} \neq \mathbf{0}$.

18. $|\mathbf{v}|$ constant $\Rightarrow a_T = \frac{d}{dt} |\mathbf{v}| = 0 \Rightarrow \mathbf{a} = a_N \mathbf{N}$ is orthogonal to $\mathbf{T} \Rightarrow$ the acceleration is normal to the path

19. $\mathbf{a} \perp \mathbf{v} \Rightarrow \mathbf{a} \perp \mathbf{T} \Rightarrow a_T = 0 \Rightarrow \frac{d}{dt} |\mathbf{v}| = 0 \Rightarrow |\mathbf{v}|$ is constant

20. $\mathbf{a}(t) = a_T \mathbf{T} + a_N \mathbf{N}$, where $a_T = \frac{d}{dt} |\mathbf{v}| = \frac{d}{dt} (10) = 0$ and $a_N = \kappa |\mathbf{v}|^2 = 100\kappa \Rightarrow \mathbf{a} = 0\mathbf{T} + 100\kappa \mathbf{N}$. Now, from

Exercise 5(a) Section 11.4, we find for $y = f(x) = x^2$ that $\kappa = \frac{|f''(x)|}{[1 + (f'(x))^2]^{3/2}} = \frac{2}{[1 + (2x)^2]^{3/2}} = \frac{2}{(1 + 4x^2)^{3/2}}$; also,

$\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j}$ is the position vector of the moving mass $\Rightarrow \mathbf{v} = \mathbf{i} + 2t\mathbf{j} \Rightarrow |\mathbf{v}| = \sqrt{1 + 4t^2}$

$\Rightarrow \mathbf{T} = \frac{1}{\sqrt{1 + 4t^2}} (\mathbf{i} + 2t\mathbf{j})$. At $(0, 0)$: $\mathbf{T}(0) = \mathbf{i}$, $\mathbf{N}(0) = \mathbf{j}$ and $\kappa(0) = 2 \Rightarrow \mathbf{F} = m\mathbf{a} = m(100\kappa)\mathbf{N} = 200m\mathbf{j}$;

At $(\sqrt{2}, 2)$: $\mathbf{T}(\sqrt{2}) = \frac{1}{3} (\mathbf{i} + 2\sqrt{2}\mathbf{j}) = \frac{1}{3}\mathbf{i} + \frac{2\sqrt{2}}{3}\mathbf{j}$, $\mathbf{N}(\sqrt{2}) = -\frac{2\sqrt{2}}{3}\mathbf{i} + \frac{1}{3}\mathbf{j}$, and $\kappa(\sqrt{2}) = \frac{2}{27} \Rightarrow \mathbf{F} = m\mathbf{a}$

$= m(100\kappa)\mathbf{N} = (\frac{200}{27} m) \left(-\frac{2\sqrt{2}}{3}\mathbf{i} + \frac{1}{3}\mathbf{j}\right) = -\frac{400\sqrt{2}}{81} m\mathbf{i} + \frac{200}{81} m\mathbf{j}$