MATH 429, LINEAR ALGEBRA

FALL 2010

1. If A is an n by m matrix, such that $n < m$, show that $AX = 0$ has a non-trivial solution.

2. Prove that an n by n matrix with a left inverse is invertible.

3. If V is a finite dimensional vector space of dimension n, then show that any set of more than n vectors in V is linearly dependent.

4. Suppose that V is a vector space. Show that any set of linearly independent vectors in V can be extended to a basis for V (you can use the result of the theorem in number 4).

5. If W_1 and W_2 are two finite-dimensional subspaces of a vector space V, show

$$\dim W_1 + \dim W_2 = \dim(W_1 \cap W_2) + \dim(W_1 + W_2).$$

6. If $\mathcal{B}$ and $\mathcal{B}'$ are two ordered bases for a vector space V, show that there is an invertible matrix P such that $[\alpha]_{\mathcal{B}'} = P[\alpha]_{\mathcal{B}}$.

7. If $T : V \to W$ is a linear transformation from a finite-dimensional vector space V to a vector space W, show that

$$\text{rank}(T) + \text{nullity(T)} = \dim V.$$

8. If $T : V \to W$ is an invertible linear transformation, then show that $\{\alpha_1, \ldots, \alpha_n\}$ is a basis for V if and only if $\{T(\alpha_1), \ldots, T(\alpha_n)\}$ is a basis for W.

9. Show that the row rank of any matrix A is equal to its column rank.