1.(1 pt) A tank contains 2540 L of pure water.A solution that contains 0.08 kg of sugar per liter enters a tank at the rate 6 L/min The solution is mixed and drains from the tank at the same rate.
(a) How much sugar is in the tank initially?
______ (kg)
(b) Find the amount of sugar in the tank after t minutes.
amount = ______
(function of t)
(c) Find the concentration of sugar in the solution in the tank after 66 minutes.
concentration = ______

2.(1 pt) A tank contains 1660 L of pure water. A solution that contains 0.03 kg of sugar per liter enters tank at the rate 9 L/min. The solution is mixed and drains from the tank at the same rate.
(a) How much sugar is in the tank at the beginning.
$y(0) =$ ______ (include units)
(b) With S representing the amount of sugar (in kg) at time t (in minutes) write a differential equation which models this situation.
$S' = f(t,S) =$

______ .

Note: Make sure you use a capital S, (and don't use S(t), it confuses the computer). Don't enter units for this function.
(c) Find the amount of sugar (in kg) after t minutes.
$S(t) =$ ______ (function of t)
(d) Find the amout of the sugar after 72 minutes.
$S(72) =$ ______ (include units)
Click here for help with units

3.(1 pt) A tank contains 2540 L of pure water. Solution that contains 0.05 kg of sugar per liter enters the tank at the rate 5 L/min, and is thoroughly mixed into it. The new solution drains out of the tank at the same rate.
(a) How much sugar is in the tank at the begining? $y(0) =$ ______ (kg)
(b) Find the amount of sugar after t minutes.
$y(t) =$ ______ (kg)
(Note that this is a function of t)
(c) As t becomes large, what value is $y(t)$ approaching ? In other words, calculate

$$\lim_{t\to\infty} y(t)$$

______ (kg)

4.(1 pt) A tank contains 100 kg of salt and 1000 L of water.A solution of a concentration 0.05 kg of salt per liter enters a tank at the rate 6 L/min. The solution is mixed and drains from the tank at the same rate.
(a) What is the concentration of our solution in the tank initially?
concentration = ______ (kg/L)
(b) Find the amount of salt in the tank after 1.5 hours.
amount = ______ (kg)
(c) Find the concentration of salt in the solution in the tank as time approaches infinity.
concentration = ______ (kg/L)

5.(1 pt) A tank contains 60 kg of salt and 2000 L of water. Pure water enters a tank at the rate 8 L/min. The solution is mixed and drains from the tank at the rate 4 L/min.
(a) What is the amount of salt in the tank initially?
amount = ______ (kg)
(b) Find the amount of salt in the tank after 1 hours.
amount = ______ (kg)
(c) Find the concentration of salt in the solution in the tank as time approaches infinity. (Assume your tank is large enough to hold all the solution.)
concentration = ______ (kg/L)

6.(1 pt) A tank contains 1440 L of pure water. A solution that contains 0.07 kg of sugar per liter enters tank at the rate 6 L/min The solution is mixed and drains from the tank at the same rate.
(a) How much sugar is in the tank at the beginning.
$y(0) =$ ______ (include units)
(b) Find the amount of sugar (in kg) after t minutes.
$y(t) =$ ______ (function of t)
(b) Find the amout of the sugar after 75 minutes.
$y(75) =$ ______ (include units)

7.(1 pt) A cell of some bacteria divides into two cells every 40 minutes.The initial population is 2 bacteria.
(a) Find the size of the population after t hours
$y(t) =$ ______
(function of t)
(b) Find the size of the population after 2 hours.
$y(2) =$ ______
(c) When will the population reach 4?
$T =$ ______

8.(1 pt) A cell of some bacteria divides into two cells every 40 minutes. The initial population is 300 bacteria.
(a) Find the population after t hours
$y(t) =$ ______ (function of t)
(b) Find the population after 9 hours.
$y(9) =$ ______
(c) When will the population reach 2700?
$T =$ ______

9.(1 pt) A bacteria culture starts with 160 bacteria and grows at a rate proportional to its size. After 3 hours there will be 480 bacteria.
(a) Express the population after t hours as a function of t.
population: ______ (function of t)
(b) What will be the population after 5 hours?

______

(c) How long will it take for the population to reach 2930 ?

10.(1 pt) A population P obeys the logistic model. It satisfies the equation
$\frac{dP}{dt} = \frac{5}{900}P(9-P)$ for $P > 0$.

(a) The population is increasing when ______ $< P <$ ______

(b) The population is decreasing when $P >$ ______

(c) Assume that $P(0) = 2$. Find $P(60)$.

______

11.(1 pt) An unknown radioactive element decays into non-radioactive substances. In 280 days the radioactivity of a sample decreases by 68 percent.

(a) What is the half-life of the element?

half-life: ______ (days)

(b) How long will it take for a sample of 100mg to decay to 76 mg?

time needed: ______ (days)

12.(1 pt) A body of mass 6 kg is projected vertically upward with an initial velocity 38 meters per second.

The gravitational constant is $g = 9.8 m/s^2$. The air resistance is equal to $k|v|$ where k is a constant.

Find a formula for the velocity at any time (in terms of k):

$v(t) =$ ______

Find the limit of this velocity for a fixed time t_0 as the air resistance coefficient k goes to 0. (Enter t_0 as t_0 .)

$v(t_0) =$ ______

How does this compare with the solution to the equation for velocity when there is no air resistance?

This illustrates an important fact, related to the fundamental theorem of ODE and called 'continuous dependence' on parameters and initial conditions. What this means is that, for a fixed time, changing the initial conditions slightly, or changing the parameters slightly, only slightly changes the value at time t.

The fact that the terminal time t under consideration is a fixed, finite number is important. If you consider 'infinite' t, or the 'final' result you may get very different answers. Consider for example a solution to y'=y, whose initial condition is essentially zero, but which might vary a bit positive or negative. If the initial condition is positive the "final" result is plus infinity, but if the initial condition is negative the final condition is negative infinity.

13.(1 pt) You have 1000 dollars in your bank account. Suppose your money is compounded every month at a rate of 0.3 percent per month.

(a) How much do you have after t years.

$y(t) =$ ______ (function of t)

(b) How much do you have after 100 months.

$y(100) =$ ______

14.(1 pt) A young person with no initial capital invests k dollars per year in a retirement account at an annual rate of return 0.06. Assume that investments are made continuously and that the return is compounded continuously.

Determine a formula for the sum $S(t)$ – (this will involve the parameter k):

$S(t) =$ ______

What value of k will provide 2922000 dollars in 46 years?

$k =$ ______

15.(1 pt) Here is a somewhat realistic example which combines the work on earlier problems. You should use the phase plane plotter to look at some solutions graphically before you start solving this problem and to compare with your analytic answers to help you find errors. You will probably be surprised to find how long it takes to get all of the details of solution of a realistic problem right, even when you know how to do each of the steps. There is partial credit on this problem.

There are 1400 dollars in the bank account at the beginning of January 1990, and money is added and withdrawn from the account at a rate which follows a sinusoidal pattern, peaking in January and in July with money being added at a rate corresponding to 1310 dollars per year, while maximum withdrawals take place at the rate of 1050 dollars per year in April and October.

The interest rate remains constant at the rate of 4 percent per year, compounded continuously.

Let $y(t)$ represents the amount of money at time t (t is in years).

$y(0) =$ ______ (dollars)

Write a formula for the rate of deposits and withdrawals (using the functions sin(), cos() and constants):

$g(t) =$ ______

The interest rate remains constant at 4 percent per year over this period of time.

With y representing the amount of money in dollars at time t (in years) write a differential equation which models this situation.

$y' = f(t,y) =$

______ .

Note: Use y rather than $y(t)$ since the latter confuses the computer. Don't enter units for this equation.

Find an equation for the amount of money in the account at time t where t is the number of years since January 1990.

$y(t) =$ ______

(c) Find the amount of money in the bank at the beginning of January 2000 (10 years later): ______

Find a solution to the equation which does not become infinite (either positive or negative) over time:

$y(t) =$ ______

During which months of the year does this non-growing solution have the highest values? $\boxed{?}$

16.(1 pt) Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between its temperature and that of its surroundings. Suppose that the temperature of a cup of coffee obeys Newton's law of cooling. If the coffee has a temperature of 210 degrees Fahrenheit when freshly poured, and 2.5 minutes later has cooled to 193 degrees in a room at 74 degrees, determine when the coffee reaches a temperature of 148 degrees.

The coffee will reach a temperature of 148 degrees in ______ minutes.

17.(1 pt) Susan finds an alien artifact in the desert, where there are temperature variations from a low in the 30s at night to a high in the 100s in the day. She is interested in how the artifact will respond to faster variations in temperature, so she kidnaps the artifact, takes it back to her lab (hotly pursued by the military police who patrol Area 51), and sticks it in an "oven" – that is, a closed box whose temperature she can control precisely.

Let $T(t)$ be the temperature of the artifact. Newton's law of cooling says that $T(t)$ changes at a rate proportional to the difference between the temperature of the environment and the temperature of the artifact. This says that there is a constant k, not dependent on time, such that $T' = k(E-T)$, where E is the temperature of the environment (the oven). Before collecting the artifact from the desert, Susan measured its temperature at a couple of times, and she has determined that for the alien artifact, $k = 0.75$.

Susan preheats her oven to 90 degrees Fahrenheit (she has stubbornly refused to join the metric world). At time $t = 0$ the oven is at exactly 90 degrees and is heating up, and the oven runs through a temperature cycle every 2π minutes, in which its temperature varies by 35 degrees above and 35 degrees below 90 degrees.

Let $E(t)$ be the temperature of the oven after t minutes.

$E(t) =$ ______

At time $t = 0$, when the artifact is at a temperature of 55 degrees, she puts it in the oven. Let $T(t)$ be the temperature of the artifact at time t. Then $T(0) =$ ______ (degrees)

Write a differential equation which models the temperature of the artifact.

$T' = f(t,T) =$

______ .

Note: Use T rather than $T(t)$ since the latter confuses the computer. Don't enter units for this equation.

Solve the differential equation. To do this, you may find it helpful to know that if a is a constant, then

$$\int \sin(t)e^{at}\,dt = \frac{1}{a^2+1}e^{at}(a\sin(t)-\cos(t))+C.$$

$T(t) =$ ______

After Susan puts in the artifact in the oven, the military police break in and take her away. Think about what happens to her artifact as $t \to \infty$ and fill in the following sentence:

For large values of t, even though the oven temperature varies between 55 and 125 degrees, the artifact varies from ______ to ______ degrees.

(To answer, you will need to use techniques you reviewed in the trig problems on this assignment to assemble two trig functions into one.)

18.(1 pt) Here is a multipart example on finance. Be patient and careful as you work on this problem. You will probably be surprised to find how long it takes to get all of the details of solution of a realistic problem right, even when you know how to do each of the steps. Use the computer to check the steps for you as you go along. There is partial credit on this problem.

A recent college graduate borrows 100000 dollars at an (annual) interest rate of 6.75 per cent. Anticipating steady salary increases, the buyer expects to make payments at a monthly rate of $875(1+t/150)$ dollars per month, where t is the number of months since the loan was made.

Let $y(t)$ be the amount of money that the graduate owes t months after the loan is made.

$y(0) =$ ______ (dollars)

With y representing the amount of money in dollars at time t (in months) write a differential equation which models this situation.

$y' = f(t,y) =$

______ .

Note: Use y rather than $y(t)$ since the latter confuses the computer. Don't enter units for this equation.

Find an equation for the amount of money owed after t months.

$y(t) =$ ______

Next we are going to think about how many months it will take until the loan is paid off. Remember that $y(t)$ is the amount that is owed after t months. The loan is paid off when $y(t) =$ ______

Once you have calculated how many months it will take to pay off the loan, give your answer as a decimal, ignoring the fact that in real life there would be a whole number of months. To do this, you should use a graphing calculator or use **a plotter on this page** to estimate the root. If you use the **the xFunctions plotter,** then once you have launched xFunctions, pull down the Multigaph Utility from the menu in the upper right hand corner, enter the function you got for y (using x as the independent variable, sorry!), choose appropriate ranges for the axes, and then eyeball a solution.

The loan will be paid off in ______ months.

If the borrower wanted the loan to be paid off in exactly 20 years, with the same payment plan as above, how much could be borrowed?

Borrowed amount = ______