Math 310 **Homework 3, Due Sept. 26, 2008**

(1) Prove:

(i): $mn = nm$ for all natural numbers m and n. (10 points)

(ii): $(mn)p = m(np)$ for all natural numbers m, n, and p. (10 points)

(2) Prove that for every natural number n, no natural number m exists such that $n < m$ and $m < n + 1$. (10 points)

(3) Let $\mathbb{N}$ be the set of natural numbers and let $S = \mathbb{N} \times \mathbb{N}$. Define a relation $\sim$ on S by setting $(x, y) \sim (u, v)$ when $xv(y + u) = yu(x + v)$.

(i): Show that $\sim$ is an equivalence relation. (10 points)

(Hint: The transitivity of $\sim$ is harder to handle. Observe, however, that $xv(y + u) = yu(x + v)$ is essentially

$$\frac{1}{u} + \frac{1}{y} = \frac{1}{v} + \frac{1}{x},$$

provided we know rational numbers; transitivity is easy to see in this setup, though it is *not* allowed at this point of our course. Can you make sense out of this in terms of natural numbers?)

(ii): List ten elements of the equivalence set of which $(1, 1)$ is a representative. (5 points)

(4)

(i): Prove the commutative law for $+$ in $\mathbb{Z}$. (5 points)

(ii): Prove the commutative law for $\cdot$ in $\mathbb{Z}$. (5 points)

(5)

(i): Prove the associative law for $+$ in $\mathbb{Z}$. (5 points)

(ii): Prove the associative law for $\cdot$ in $\mathbb{Z}$. (5 points)

(6) Prove the distributive law in $\mathbb{Z}$. That is, show that for all integers $a, b, c \in \mathbb{Z}$, we have $a \cdot (b + c) = a \cdot b + a \cdot c$. (10 points)