

### Math 310 Homework 4, Due Oct 3, 2008

(1) Prove that multiplication for  $\mathbb{Z}$  is well-defined independent of the representatives chosen. That is, show that if  $[a, b] = [c, d]$  and  $[e, f] = [g, h]$  are integers, then  $[a, b] \cdot [e, f] = [c, d] \cdot [g, h]$ . (5 points)

(2) Use ONLY the definition of addition and multiplication for  $\mathbb{Z}$  to show that the cancellation laws hold true for equality. That is, show that for all  $x, y, z \in \mathbb{Z}$ , we have

- (a)  $x + y = x + z$  implies  $y = z$  (5 points),
- (b)  $xy = xz$  with  $x \neq 0$  implies  $y = z$  (10 points).

(3) Use ONLY the trichotomy law for integers to prove the cancellation law for inequality. That is, assume  $xz \leq yz$  for  $x, y, z \in \mathbb{Z}$ .

- (1) If  $z > 0$  then  $x \leq y$ . (5 points)
- (2) If  $z < 0$  then  $y \leq x$ . (5 points)

(4) Show that addition for  $\mathbb{Q}$  is well-defined independent of the representatives chosen. (5 points)