Covariance Problems

1) Suppose that $X, Y$ are RV's with $\text{Var} \ X = 9$, $\text{Var} \ Y = 4$, and $\rho(X,Y) = -\frac{1}{6}$. Find
   (a) $\text{Var} \ (X + Y)$, (b) $\text{Var} \ (X - 3Y + 4)$, (c) $\text{Cov} \ (X, Y)$

2) Suppose that $X, Y, Z$ are RV's with
   $\text{Var} \ X = 1$, $\text{Var} \ Y = 4$, $\text{Var} \ Z = 8$,
   $\text{Cov} \ (X, Z) = -1$, $\text{Cov} \ (X, Y) = 1$, $\text{Cov} \ (Y, Z) = 2$. Find
   (a) $\text{Cov} \ (X+Y, X+Z)$ (b) $\text{Var} \ (3X - Y - 2Z + 1)$

3) Suppose that $X_1, \ldots, X_n$ are RV's s.t. the variance of each RV is 1 and the correlation between each pair of different variables is $\frac{1}{4}$. Find
   $\text{Var} \left( \sum_{i=1}^{n} X_i \right)$.

4) If $\text{Var} \ X = 9$, $\text{Var} \ Y = 4$, use Schwarz's inequality to find the largest and smallest possible values of $\text{Cov} \ (X, Y)$. 
5) Suppose that discrete RV's have p.f. specified by the table

<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>4</th>
<th>5</th>
<th>6</th>
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<tbody>
<tr>
<td>1</td>
<td>.3</td>
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<td>2</td>
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Find $E(XY)$ and $\text{Cov}(X,Y)$ and $f(x,y)$.

Answers

1) (a) 11, (b) 51, (c) -1

2) (a) 3, (b) 59

3) $n + \frac{n(n-1)}{4}$

4) Largest = 6, Smallest = -6

5) $E(XY) = 6.6$, $EX = 1.4$, $EY = 4.7$

$\text{Cov}(X,Y) = .02$

$E(X^2) = 2.2$, $\text{Var} X = .24$

$E(Y^2) = 23.7$, $\text{Var} Y = .61$

$f(x,y) = .052$