1. \( f(x) = \frac{3}{7} x^2, \quad 1 < x < 2 \)

(a) \( F(x) = 0, \quad x \leq 1 \)

\[ = \begin{cases} 1 & x \geq 2 \\ \frac{1}{7} (x^3 - 1) & 1 < x < 2 \end{cases} \]

(b) \( p_X (1.5 < X < 3) = \int_{1.5}^{3} \frac{3}{7} x^2 \, dx = \frac{3}{7} \int_{x=1}^{x=3} x^2 \, dx = \frac{3}{7} \left[ \frac{3}{5} x^5 \right]_{1}^{3} = \frac{3 \times 15}{28} = \frac{45}{28} \)

(c) \( E[X] = \int_{1}^{3} \frac{2}{7} x^3 \, dx = \frac{2}{28} x^4 \left| \right. x=3 | x=1 = \frac{3}{28} = \frac{45}{28} \)

(2) \( E\left[\frac{1}{X^2}\right] = \int_{1}^{3} \frac{1}{x^2} \frac{3}{7} x^2 \, dx = \frac{3}{7} \)

2. \( f = 6y \quad \text{in} \quad T \)

(a) \( P_U (X + Y < 2) \)

\[ = \int_{0}^{2} \int_{0}^{2-x} 6y \, dy \, dx \]

\[ = \int_{0}^{2} 3 \left( \frac{1}{2} - x \right)^2 \, dx \]
\( g_2(y|x) = \frac{f(x, y)}{\varphi_1(x)} \)

\( \varphi_1(x) = 3(1-x)^2 \)

\( \varphi_1(\frac{2}{3}) = \frac{1}{3} \)

\( g_2(y|\ x = \frac{2}{3}) = \frac{6y}{1/3}, \quad 0 \leq y \leq \frac{1}{3} \)

\[ = 0, \quad \text{otherwise} \]

\[ \mathbb{P}(Y \geq \frac{1}{4} \mid x = \frac{2}{3}) = \int_{\frac{1}{4}}^{\frac{1}{3}} 1 \ dy \ d\ y. \]
small box centered at \((\frac{1}{3}, \frac{1}{4})\) is approximately \(\frac{3}{4}\) times the area of the box.

\[ F(1) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 0.9 \]

\[ P_n(\chi = 1) = 0 \]

\[ P_n(\chi = 2) = 0.8 - 0.2 = 0.6 \]

\[ P_n(2 \leq \chi \leq 3) = 0.9 - 0.2 = 0.7 \]

\[ y = (\chi - 1)^{\frac{1}{2}} \]

\[ p(x) = \frac{3}{2} x^2 \text{ on } (1, 2), \text{ range } \chi = (0, 1) \]

\[ P_n(Y \leq y) = P_n\left( (\chi - 1) \leq y \right) \]

\[ = P_n\left( \chi - 1 \leq y^2 \right) \quad [\text{since } \chi \geq 1] \]

\[ = P_n\left( \chi \leq 1 + y^2 \right) \]

\[ = F(1 + y^2), \quad 0 \leq y \leq 1 \]

\[ g(y) = g'(y) = 2y \frac{1}{2} = \frac{1}{2}(1 + y^2) \]

\[ = \frac{1}{2} y^2 - \frac{3}{7} (1 + y^2)^2 \]

\[ = \frac{1}{7}y^2(1 + y^2)^2, \quad 0 < y < 1 \]

\[ g(y) = 0 \quad \text{for } \chi > 0 \text{ then } y, \]

\[ P_n\left( \min (X_1, X_2) \geq t \right) = P_n\left( X_1 \geq t \text{ and } X_2 \geq t \right) \]

\[ = [P_n(X_1 \geq t)]^2 = (1 - F(t))^2 = (1 - \frac{1}{7}(1 - t^2))^2, \quad 1 < t < 2. \]

\[ = \left( \frac{3 - t^2}{4} \right)^2 \]
\[ h(x) = \int_{-\infty}^{\infty} f(w) f(x-w) \, dw, \quad \text{where} \quad f = \phi d + \psi + \mathbb{Z}, \]

\[ = \int_{x-1}^{x} f(w) \, dw, \quad \text{as in class and HW C}, \]

\[ = \int_{x-1}^{x} 2e^{-iw} \, dw, \quad \text{when} \quad x > 1 \]

\[ = -e^{-iw} \bigg|_{w=x-1}^{w=x} \]

\[ = e^{-2(x-1)} - e^{-2x}, \quad \text{when} \quad x > 1. \]

6. \[ y_1 = 2x_1, \quad y_2 = 2x_1 + 4x_2. \]

\[ x_1 = \frac{y_1}{2}, \quad 4x_2 = y_2 - 2x_1 = y_2 - y_1, \]

\[ x_2 = \frac{y_2 - y_1}{4}. \]

If \( y_1 = 1, \ y_2 = 2, \) then \( x_1 = \frac{1}{2}, \ x_2 = \frac{1}{4}. \)

\[ g(y_1, y_2) = \frac{1}{|\det A|} \quad \rho(\frac{1}{2}, \frac{1}{4}). \]

\[ \det A = \det \begin{vmatrix} 2 & 0 \\ 2 & 4 \end{vmatrix} = 8 \]

\[ \rho(\frac{1}{2}, \frac{1}{4}) = \frac{3}{2} \]

\[ g(1, 2) = \frac{1}{8} \cdot \frac{3}{2} = \frac{3}{16}. \]