Math 493.  Homework 9.  Due Dec 2, 2005

1. (a) Do Problem 8, p.251. Let $X$ be the number of failures.

(b) Find the unconditional probability that at least two components fail.

(c) Find the conditional mean of the number of failures, given that at least one component failed.

Note that the unconditional mean is $EX = 2$.

2. Suppose that 5 persons are selected at random to go shopping from a group consisting of 6 men and 8 women, Let $X$ be the number of women selected. Find

(a) $P(X \leq 2)$  (b) $EX$  (c) $VarX$.

(d) Suppose that we independently choose a group of 5 shoppers on each of 50 successive days. Let $\bar{X}_{50}$ be the average number of women selected. Use Chebyshev’s inequality to get a lower bound on the probability that the sample mean $\bar{X}_{50}$ will differ from the theoretical mean $EX$ by at most 0.3

3. (a) Do Problem 14, p.263. Let $X =$ number of no-shows.

(b) Suppose that the plane has $N$ seats, where $N < 198$. Find the smallest $N$ for which the approximate probability that all passengers who show up will get seats is at least 0.1.

You should work 3a two different ways: Find the approximate probability using a Poisson approximation, and find the exact probability using a binomial distribution. For 3b, use only the Poisson approximation.
4. Problem 2, p. 267. And add a part (c): What is the probability that the 5'th head occurs on the twelfth toss?

R1. Problem 8, p.235. In each of (a) and (b) find the smallest $n$ which works. Be sure to state the mean and variance of $Q_n$. And is it true or false, in each of the parts, that if some $n$ works then all larger $n$'s work?

R2. Problem 8, p.255. You can let $X_i$ be the r.v. with $X_i = a_i$ if the i'th person is selected, $X_i = 0$ if he is not.

R3. Problems from DeGroot, especially about Poisson and negative binomial.