Let $B$ = the diameter of the selected bolt, & let $N$ = the diameter of the selected nut.

Then $B \sim N(8.5, (0.04)^2)$ & $N \sim N(2.53, (0.02)^2)$.

The nut & bolt will fit together if $N > B$ & $N - B \leq 0.05$.

So $Pr(\text{nut & bolt fit together}) = Pr(0 < N - B \leq 0.05)$

Note $N - B \sim N(0.03, 0.002^2)$, so $Z = \frac{N - B - 0.03}{\sqrt{0.002}} \sim N(0, 1)$

Now $Pr(0 < N - B \leq 0.05) = Pr\left(\frac{0.03}{\sqrt{0.002}} < Z \leq \frac{0.02}{\sqrt{0.002}}\right)$

$= Pr(Z \leq \frac{0.02}{\sqrt{0.002}}) - Pr(Z \leq \frac{0.03}{\sqrt{0.002}})$

$= \Phi\left(\frac{0.02}{\sqrt{0.002}}\right) - \left(1 - \Phi\left(\frac{0.03}{\sqrt{0.002}}\right)\right)$

$\approx 0.6736 - (1 - 0.7486)$

$= 0.4222$

So the probability that the nut & bolt will fit together is 0.4222.
2a) Let $A =$ the score of a student from University A, and let $B =$ the score of a student from University B.

Then $A \sim N(605, 50)$ and $B \sim N(600, 50)$.

We want $Pr(\bar{B} - \bar{A} > 0)$.

Note $\bar{A} \sim N(605, 50)$ and $\bar{B} \sim N(600, 50)$.

So $\bar{B} - \bar{A} \sim N(-5, 100)$.

Let $Z = \frac{\bar{B} - \bar{A} + 5}{10}$, so $Z \sim N(0, 1)$.

So the probability that the average of 2 students from A will be greater than the average of 3 from B is 0.6915.

2b) Let $p_n = Pr(\bar{B}_n - \bar{A}_n < 0)$. Want $p_n > 0.999$.

Let $Z_n = \frac{\bar{B}_n - \bar{A}_n + 5}{\sqrt{100 + 150}}$, so $Z_n \sim N(0, 1)$.

Then $Pr(\bar{B}_n - \bar{A}_n < 0) = Pr(Z_n < \frac{5}{\sqrt{250}}) = Pr(Z_n < \frac{5\sqrt{n}}{\sqrt{250}})$.

From the table, $Pr(Z_n < 5\sqrt{n/250}) > 0.999$ when $5\sqrt{n/250} > 3.10$.

So when $n = 109$, we see that $p_n > 0.999$ is satisfied.
3a) \( X \sim B(20, \frac{1}{2}) \), \( Q = \Pr(8 \leq X \leq 12) \). Find the exact value of \( Q \).

\[
\Pr(8 \leq X \leq 12) = \Pr(X=8) + \Pr(X=9) + \Pr(X=10) + \Pr(X=11) + \Pr(X=12)
\]

So \( Q = \left[ \binom{20}{8} + \binom{20}{9} + \binom{20}{10} + \binom{11}{11} + \binom{12}{12} \right] \left(\frac{1}{2}\right)^{20} \approx 0.7368 \)

3b) Let \( Z = \frac{X - \mu}{\sigma} = \frac{X - 10}{\sqrt{20(\frac{1}{2})(\frac{1}{2})}} \), so \( Z \sim N(0,1) \)

Then \( \Pr(8 \leq X \leq 12) = \Pr(-\frac{3}{\sqrt{5}} \leq Z \leq \frac{2}{\sqrt{5}}) \)

\[
= \Pr(1 \leq 1 \leq \frac{2}{\sqrt{5}})
\]

\[
= 2 \Phi\left(\frac{2}{\sqrt{5}}\right) - 1
\]

\( \approx 2(0.8133) - 1 = 0.6266 \)

So \( Q \approx 0.6266 \)

3c) With the correction for continuity we want

\( \Pr(7.5 \leq X \leq 12.5) = \Pr\left(-\frac{7.5-10}{\sqrt{20(\frac{1}{2})(\frac{1}{2})}} \leq Z \leq \frac{12.5-10}{\sqrt{20(\frac{1}{2})(\frac{1}{2})}}\right) \)

\[
= \Pr\left(1 \leq 1 \leq \frac{2.5}{\sqrt{5}}\right)
\]

\[
= 2 \Phi\left(\frac{2.5}{\sqrt{5}}\right) - 1
\]

\( \approx 2(0.8023) - 1 = 0.7372 \)

So \( Q \approx 0.7372 \)
4a. Let $X_i$ = size of drink $i$, with $\mu_i = 2$ and $\sigma_i = \frac{1}{2}$.

Let $X = \frac{\sum_{i=1}^{30} X_i}{30}$, so $X \sim N(72,9)$.

Let $Z = \frac{X - 72}{9}$, so $Z \sim N(0,1)$.

The probability that the bottle will supply 30 is

$\Pr(X \leq 69) = \Pr\left(Z \leq \frac{69 - 72}{9}\right) = \Pr(Z \leq -3) \approx 1 - 0.9987$. \checkmark

Now let $S$ = size of the bottle.

We want $\Pr(X \leq S) = \frac{1}{2}$, or $\Pr\left(Z \leq \frac{S - 72}{9}\right) = \frac{1}{2}$.

So we need $\frac{S - 72}{9} = 0$, i.e. $S = 72$.

So the bottle should be 72 ounces.

4b. Now we want $\Pr(X \leq S) = 0.99$, or $\Pr\left(Z \leq \frac{S - 72}{9}\right) = 0.99$.

From the table we see that we need $\frac{S - 72}{9} = 2.33$,

so $S = 78.99$.

So the bottle should be 78.99 ounces.
Now we suppose \( \mu_i = 2 \) \& \( \sigma_i = 0.1 \).

Then \( X = \sum_{i=1}^{35} X_i \sim N(72, 0.36) \).

Let \( Z = \frac{X - 72}{0.6} \), so \( Z \sim N(0, 1) \).

Then \( \Pr(X \leq 8) = \frac{1}{2} \iff \Pr\left(Z \leq \frac{8 - 72}{0.6}\right) = \frac{1}{2} \)

\[ \iff \frac{8 - 72}{0.6} = 0 \]

\[ \iff S = 72. \]

So again, the bottle should be 72 ounces for the probability of its being nonempty after 36 drinks to be \( \frac{1}{2} \).

Now \( \Pr(X \leq 8) = 0.99 \iff \Pr\left(Z \leq \frac{8 - 72}{0.6}\right) = 0.99 \)

\[ \iff \frac{8 - 72}{0.6} = 2.33 \]

\[ \iff S = 72.398. \]
Let $p_i =$ probability girl $i$ hits target.

So $p_a = 0.3$, $p_b = 0.2$, & $p_c = 0.1$

Let $X =$ total number of target hits, & let $y_i =$ number of times girl $i$ throws.

So $y_a = 10$, $y_b = 15$, & $y_c = 20$.

Now $E(X) = p_a y_a + p_b y_b + p_c y_c$

$= (0.3)(10) + (0.2)(15) + (0.1)(20) = 8$ &

$Var(X) = p_a y_a^2 + p_b y_b^2 + p_c y_c^2$

$= (0.3)(100) + (0.2)(225) + (0.1)(400) = 10.3$

Now $Pr$(target hit at least 12 times) = $Pr$(X $\geq$ 12).

So let $Z = \frac{X - 8}{\sqrt{10.3}}$, so $Z \sim N(0, 1)$

Then $Pr(X \geq 12) = Pr(Z \geq \frac{4}{\sqrt{10.3}}) = 1 - \Phi\left(\frac{4}{\sqrt{10.3}}\right) \approx 1 - 0.9441$

So $Pr$(target hit at least 12 times) $\approx 0.0559$

With correction for continuity, we have

$Pr(X \geq 12) = Pr(Z \geq 5) = Pr(Z \geq \frac{5}{\sqrt{10.3}}) = 1 - \Phi\left(\frac{5}{\sqrt{10.3}}\right) \approx 0.0883$