1. \( S = \) all 5 element subsets of \( \{1, \ldots, 52\} \)

\[ |S| = \binom{52}{5}. \]

Let \( A = \) flushes, \( B = \) straights, \( C = \) 4 of a kind,

\[ |A| = 4 \cdot \binom{13}{5}, \quad |B| = 9 \cdot 4 \cdot 5, \quad |C| = 13 \cdot 4 \cdot 8 \]

(a) \( P(A) = \frac{9 \cdot 4 \cdot 5}{\binom{52}{5}} = \frac{92}{54,145} = .00355 \)

(b) \( P(A) = 4 \cdot \frac{\binom{13}{5}}{\binom{48}{5}} = \frac{33}{16,660} = .00198 \)

(c) \( P(C) = \frac{13 \cdot 4 \cdot 8}{\binom{52}{5}} = \frac{1}{4165} = .00024 \)

2. (a) \( \binom{14}{5} = 2002 \)

(b) \# of boy teams = \( \binom{8}{5} \)

\( P(\text{all boys}) = \frac{\binom{8}{5}}{\binom{14}{5}} = \frac{4}{143} = .02797 \)

(c) \# with 3 boys, 2 girls = \( \binom{8}{3} \cdot \binom{6}{2} \)

\( P(\text{3 boys, 2 girls}) = \binom{8}{3} \cdot \frac{1}{8} = \frac{60}{143} = .4196 \)

(d) \( P(\text{E and F on same team}) = P(\text{both on A}) + P(\text{both on B}) \)

\[ = \frac{\binom{12}{3}}{\binom{14}{5}} + \frac{\binom{12}{7}}{\binom{14}{5}} = \frac{46}{91} = .505 \]
(e) Let $S = \text{all } 1-1 \text{ functions from } \{1, \ldots, 5\} \text{ into } \{1, \ldots, 14\}$, and let $\mathcal{B} = \{1, \ldots, 8\}$ be boys. Let $A = \text{ element } \pi \text{ of } S$ with

$$
\pi(1), \pi(2), \pi(3) \text{ in } \mathcal{B}, \quad \pi(4), \pi(5) \notin \mathcal{B}.
$$

Then

$$|A| = \frac{3! \cdot 2!}{5!} = \frac{6}{120} = 0.0416667 = \frac{8! \cdot 9!}{5! \cdot 14!}.$$

(3) (a) $P_n(\text{GBW}) = \frac{7 \cdot 10 \cdot 6}{23^3} = 0.0416667$, $P_n(3 \text{ different colors})$,

$$= 3! \cdot P_n(\text{GBW}) = \frac{7 \cdot 10 \cdot 6}{23^3} \cdot 6 = 0.20712.$$

(b) Can be done by calculating $P_n(\text{GBB})$, $P_n(\text{GGW})$, etc., then adding. But it's easier to use the fact that

$$P_n(\text{exactly 2}) = 1 - P_n(\text{1 color appears}) - P_n(\text{3 colors appear})$$

$$= 1 - 0.66 - 0.20712 = 0.118818.$$

(c) $P_n(\text{only one color appears}) = P_n(\text{BBB}) + P_n(\text{WW}) + P_n(\text{GGG})$

$$= \frac{1}{23^3} \left[ 7^3 + 6^3 + 10^3 \right] = \frac{1559}{23^3} = 0.188.$$

(4) Let $S = \text{all partitions of } \{1, \ldots, 23\} \text{ into sets}$

of size 1, 6, 5, 3, 4, 10. These represent chores, etc.
(a) Let $A =$ event "all $p$'s on 2nd night." Then

$$|A| = \binom{17}{11, 1, 5}.$$ 

$$P_n(A) = \frac{|A|}{\binom{22}{11, 6, 5}} = \frac{\binom{6}{5}}{\binom{22}{5}} = \frac{1}{4387} = 2.28 \times 10^{-4}$$

(b) Let $B =$ this event. Then $|B| = \binom{10}{6, 2, 2} \binom{12}{5, 4, 3}$

$$P_n(B) = \frac{|B|}{|S|} = .1072$$

$$\binom{10}{6, 2, 2} = 1,260, \quad \binom{12}{5, 4, 3} = 27,720$$

$$\binom{22}{11, 6, 5} = 325,909,584$$

$$P_n(B) = .1072$$