
1. The exact probability that 100 tosses of a fair coin give exactly 50 heads is .0796. Use the CLT to find an approximate probability,

2. If \( X \sim N(\mu, \sigma^2) \), then \( P(|X - \mu| \leq \sigma) = .6826 \). This is sometimes expressed by saying that approximately 68.26% of the measurements of \( X \) should lie within 1 standard deviation of the mean. What percentage should lie between 2 s.d.'s of the mean? Within 3?

3. Use moment generating functions to find \( E(X^4) \) when \( X \) has a chi-square distribution with 3 degrees of freedom.

4. Suppose that the number of minutes \( X \) needed to serve a customer at the checkout counter of a supermarket has an exponential distribution for which the mean is 5 minutes. Thus, the parameter \( \beta \) for \( X \) is 1/5.

   (a) Find \( Pr(X > 7) \).

   (b) Find the median time it takes to get served.

   (c) If you have been waiting 10 minutes and have not yet been served, what is the probability that you will have to wait at least 7 more minutes?

   (d) Let \( Y \) be the total time it takes for one checker to serve a random sample of 20 customers. Then \( Y \) has a \( \chi^2 \) distribution with parameters **. What are \( !! \) and **?

   (e) Find the mean and variance of \( Y \).

   (f) Write down a definite integral which gives the probability that \( Y \) is larger than two hours. Estimate this integral numerically, if you can.
(g) Use the CLT to find an approximate value for the probability in (f).

5. Suppose that each driver from some homogeneous pool is assumed to have a probability $p$ of having at least one accident in 5 years, and that $p$ is a r.v. with a beta distribution with parameters $\alpha = 3.5$ and $\beta = 2$.

(a) If a driver is selected at random, what is the probability that his accident probability is $< 1/4$? Is $> 3/4$? One of these probabilities is larger than the other. Draw a graph of the beta$(3.5, 2)$ p.d.f. which illustrates why this so.

(b) If two drivers are independently selected at random, what is the probability that the larger of the two $p$’s will be at least $1/2$?

(c) If $p$ really is a beta $(3.5, 2)$ the highways would likely be too dangerous to travel. For example, the mean accident probability for drivers in the pool would be $.636$. Let's be sanguine instead of sanguinary and suppose that $p$ has a beta distribution whose mean is $.1$ and variance is $.01$. What are the parameters $\alpha$ and $\beta$?

6. Suppose that $X$ and $Y$ are independent standard normal r.v.'s. Then, in appropriate units, $(X, Y)$ represents the position at some fixed time $t$ of a particle in the plane which executes a two-dimensional Brownian motion starting at the origin. $R = (X^2 + Y^2)^{1/2}$ is its distance from the origin at time $t$. $R$ is said to have a Rayleigh distribution.

(a) Compute the pdf of $R$. Suggestion: $R^2$ is one of the gammas, so we have a formula for its p.d.f.

(b) Find the d.f. of $R$, and also $Pr(R \leq 1)$.

From $D \in \mathcal{G}$ note:

<table>
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<th>5.9</th>
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<tr>
<td>5.12</td>
<td>1, 2, 3, 5, 7, 11</td>
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</table>
Answers to PP 2

1. 0.796  2. 95.46  0.70, 99.74  0.70

3. 945  4. (a) 2.466, (b) 3.4666, (c) 2.466, (d) \( \text{gamma}(20, \frac{1}{5}) \)
   (e) \( 100, 500 \) (f) \( 5^{-20} \frac{1}{19!} \int_{120}^{\infty} x^{19} e^{-x/5} \, dx \)
   (g) 0.1867

5. (a) 0.0283, 0.315, (b) 0.9409
   (c) \( d = 0.8, \beta = 7.2 \)

6. (a) \( f(y) = ye^{-\frac{1}{2}y^2}, \quad y > 0 \)
   \( = 0, \quad y < 0 \)
   (b) \( g(y) = 1 - e^{-\frac{1}{2}y^2}, \quad y > 0 \)
   \( = 0, \quad y < 0 \)
   \( \Pr(R \leq 1) = 1 - e^{-\frac{1}{2}} = 0.393 \)