1. With the data of HW 7, find a 95% confidence ellipse for \((\beta_0, \beta_1)\). Sketch its graph, and indicate on your graph the center, the lengths of the major and minor axes, and the angle the major axis makes with the \(\beta_0\) axis in the \(\beta_0\beta_1\) plane. Decide on the basis of the p-value in Problem 4 of HW 7 whether \((-2, 0)\) should be inside or outside the ellipse. Then test your decision by substituting \((-2, 0)\) into the inequality defining the interior of the ellipse.

2. Consider a normal homoscedastic bilinear regression model

\[
y_i = \beta_0 + \beta_1 x_{i,1} + \beta_2 x_{i,2} + \epsilon_i,
\]

and suppose that we have the following data:

\[
\begin{bmatrix}
x_1 & -1 & 0 & 1 & -1 & 0 & 1 \\
x_2 & 1 & 0 & 0 & 0 & 1 & 1 \\
y & 2 & 1 & 1.5 & .5 & 2 & 5
\end{bmatrix}
\]

(a) Find the covariance matrix of the rv's \(\hat{\beta}_i, i = 0, 1, 2\). First, you should write down the design matrix \(Z\).

(b) Use the covariance matrix to calculate \(\text{Var}(a \hat{\beta}_0 + b \hat{\beta}_2)\), where \(a\) and \(b\) are constants.

(c) Calculate the \(\hat{\beta}_i\) using the given data. Then write down the equation of the estimated regression function of \(y\) on \(x_1\) and \(x_2\).

(d) Estimate the variance \(\sigma^2\) of the \(Y_i\), using an unbiased estimator.

(e) Find the estimated average value of \(Y\) when \(x_1 = 2\) and \(x_2 = 3\).

(f) What proportion of the variation is "explained" by the fitted regression? See p. 658.
Note. In Problem 2 the value of the statistic \( S^2 \) is 2.5, as you should verify.

3. With the data of Problem 2, test each of the following null hypotheses at significance level .05.
   
   (a) \( \beta_2 = 0 \).  (b) \( \beta_1 = \beta_2 = 0 \).  (c) \( \beta_0 = \beta_1 = \beta_2 = 0 \).

4. Find an appropriate confidence set for each part of Problem 3, with confidence coefficients .95. Make sure your results are consistent with acceptance or rejection of the null hypotheses in Problem 3.