1. We'll proceed by induction. Let $S = \{ n \in \mathbb{N} : n \neq n' \}$. 
   First, note that $1 \in S$. If it weren't, $1 = 1'$, so $1$ would be the successor of something, violating Peano Axiom #4.

   Next, suppose $n \in S$. We want to show $n' \notin S$.
   Suppose $n' \notin S$. Then $(n')' = n'$, and so, by Peano Axiom #3, $n' = n$, a contradiction.

   Since we have shown $1 \in S$ and that $n \in S \implies n' \notin S$, we may conclude that $S = \mathbb{N}$ by PA #5.

2. Fix $m \in \mathbb{N}$. Let $S = \{ n \in \mathbb{N} : m + n \neq n' \}$. 
   First, note that $1 \in S$. If it wasn't, $m + 1 = 1$, and so $m' = 1$, violating PA #4.

   Suppose $n \in S$. In anticipation of a contradiction, suppose $n' \notin S$; i.e., that $m + n' = n'$. But $m + n' = (m + n)'$, and so, by PA #3, $m + n = n$, a contradiction. Thus, $n' \notin S$.

   By induction, we may conclude $S = \mathbb{N}$. Since our choice of $m$ was arbitrary, we have that $m + n \neq n$ for all $m, n \in \mathbb{N}$.

3. Fix $m \in \mathbb{N}$. Let $S = \{ n \in \mathbb{N} : m \cdot n \neq n' \}$. 
   First, note that $1 \in S$. If it wasn't, we would have $m \cdot 1 = 1$, and so $m' = 1$, contradiction PA #4.

   Suppose $n \in S$. In anticipation of a contradiction, suppose $n' \notin S$. Then $m \cdot n' = 1$, so $(m \cdot n)' = 1$ by Theorem 1. 
   However, this violates PA #4, so $n' \in S$.

   By induction, $S = \mathbb{N}$, and since $m$ was arbitrary, we have that $m \cdot n \neq 1$ for all $m, n \in \mathbb{N}$.
4. Fix \( m, n \in \mathbb{N} \) such that \( m \neq n \). Let \( S = \{ p \in \mathbb{N} : m + p \neq n + p \} \).

First, \( 1 \in S \). If it wasn't, \( m1 = n1 \)

\[ \Rightarrow m' = n' \]

\[ \Rightarrow m = n \quad \text{by PA \#3} \]

But \( m \neq n \). Thus, \( 1 \in S \).

Now suppose \( p \in S \). In anticipation of a contradiction, suppose \( p' \not\in S \). Then:

\[ m + p' = n + p' \]

\[ \Rightarrow (m + p)' = (n + p)' \quad \text{by Theorem 1} \]

\[ \Rightarrow m + p = n + p \quad \text{by PA \#3} \]

But that contradicts the fact that \( p \in S \). Therefore, \( p' \in S \), and so by induction, \( S = \mathbb{N} \).

Our choice of \( m, n \in \mathbb{N} \), \( m \neq n \), was arbitrary, and so \( m + p \neq n + p \) for all \( m, n, p \in \mathbb{N} \).