1) A mass on a spring vibrates vertically with its position at time $t$ given by $y(t) = \cos(t) + \sin(t)$, $0 \leq t \leq 2\pi$.

At what time will its acceleration be zero?

**Solution:** Acceleration is the second derivative of the position function.

$$y(t) = \cos(t) + \sin(t), \quad (\cos(t))' = -\sin(t), \quad (\sin(t))' = \cos(t)$$

$$\therefore y'(t) = -\sin(t) + \cos(t)$$

$$y''(t) = -\cos(t) - \sin(t), \text{ this is the function of acceleration}$$

$$\therefore y''(t) = 0 \Rightarrow -\cos(t) - \sin(t) = 0$$

$$\Rightarrow \sin(t) = -\cos(t)$$

$$\Rightarrow \tan(t) = -1, \quad 0 \leq t \leq 2\pi$$

$$\Rightarrow t = \frac{3\pi}{4} \text{ and } t = \frac{7\pi}{4}$$

(over)
2) Find an equation for the tangent line to the curve \( y = \frac{x}{1+e^x} \) at the point \((0,0)\).

**Solution:** The slope of the tangent line is the derivative.

\[
y = \frac{x}{1+e^x},
\]

by quotient rule,

\[
y' = \frac{(1+e^x)-x(e^x)}{(1+e^x)^2} = \frac{1+e^x-xe^x}{(1+e^x)^2}.
\]

At point \((0,0)\),

the slope \( m = y' \bigg|_{x=0} = \frac{1+0-0}{(1+0)^2} = \frac{1+1}{(1+1)^2} = \frac{2}{4} = \frac{1}{2} \).

\[
\therefore \text{ The tangent line is } y - 0 = \frac{1}{2} (x - 0)
\]

\[
\therefore y = \frac{1}{2} x
\]