1) For the function \( f(x) = 4x^3 - x^4 \) find out where it is increasing and where it is concave down.

Solution: (i) \( f'(x) = 12x^2 - 4x^3 \)
\[ f'(x) = 0 \Rightarrow 12x^2 - 4x^3 = 0 \Rightarrow 4x^2(3-x) = 0 \Rightarrow x = 0 \text{ or } x = 3. \]
\( \therefore \) The critical points are \( x = 0 \) and \( x = 3. \)

When \( x < 0, f'(x) > 0, \) increasing: \( + + + + + + + + + + + + \)
When \( 0 < x < 3, f'(x) > 0, \) increasing: \( + + + + + + + + + + + + \)
When \( x > 3, f'(x) < 0, \) decreasing: \( - - - - - - - - - - - - \)

\[ \begin{array}{c|cccccc}
    & 0 & 1 & 2 & 3 \\
\hline
    f'(x) & + & + & + & + & + & + \\
\end{array} \]

(ii) \( f''(x) = 24x - 12x^2 \)
\[ f''(x) = 0 \Rightarrow 24x - 12x^2 = 0 \Rightarrow 12x(2-x) = 0 \Rightarrow x = 0 \text{ or } x = 2. \]

When \( x < 0, f''(x) < 0, \) concave down,
When \( 0 < x < 2, f''(x) > 0, \) concave up,
When \( x > 2, f''(x) < 0, \) concave down.

\[ \begin{array}{c|ccccc}
    & - & - & - & + & + & + & + \\\n    & down & up & 0 & down & 2 & \text{(over)} & down \end{array} \]
a) Sketch a graph of the function on the other side (problem #1) showing all local max, local min and inflection points.

\[ f(3) = 4(3)^3 - 3^4 = 4 \cdot 27 - 81 = 27, \]
\[ f(0) = 0, \]
\[ f(2) = 4(2)^3 - 2^4 = 4 \cdot 8 - 16 = 16. \]

Inflection points: (0, 0), (2, 16).

When \( x = 3 \), \( f(x) \) has local max 27.