1) Use Midpoint rule to approximate
\[ \int_{1}^{4} x^3 \, dx, \quad n = 6. \]

So, our subintervals are:

\[ [1, 1 + \Delta x] = [1, 1.5] \]
\[ [1.5, 1.5 + \Delta x] = [1.5, 2] \]
\[ [2, 2 + \Delta x] = [2, 2.5] \]
\[ [2.5, 3] \]
\[ [3, 3.5] \]
\[ [3.5, 4] \]

The midpoints of these intervals are 1.25, 1.75, 2.25, 2.75, 3.25, 3.75. Thus

\[ M_6 = f(1.25)\Delta x + f(1.75)\Delta x + f(2.25)\Delta x + f(2.75)\Delta x + f(3.25)\Delta x + f(3.75)\Delta x \]

\[ = (1.25)^3(1.5) + (1.75)^3(1) + (2.25)^3(1.5) + (2.75)^3(1) + (3.25)^3(1.5) + (3.75)^3(1) \]

\[ \approx 63.28 \]

Note: The exact answer is
\[ \int_{1}^{4} x^3 \, dx = \left[ \frac{x^4}{4} \right]_{1}^{4} = \frac{4^4}{4} - \frac{1^4}{4} = 04 - \frac{1}{4} = 63.75 \]

\[ M_6 \] is a close estimate.
2) Evaluate \( \int_1^4 \frac{1}{2} x + 1 \, dx \)

by interpreting it in terms of area.

The area under the graph of \( \frac{1}{2} x + 1 \) is composed of a rectangle & a triangle. So, \( \int_1^4 \frac{1}{2} x + 1 \, dx \) equals the area of the rectangle plus the area of the triangle.

\[
\text{area}_{\text{rectangle}} = \text{length} \cdot \text{height} = (4-1)(1.5) = 9/2
\]

\[
\text{area}_{\text{triangle}} = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} (4-1)(3-1.5) = 9/4
\]

\[
\int_1^4 \frac{1}{2} x + 1 \, dx = \frac{9}{2} + \frac{9}{4} = 27/4 = 6 \frac{3}{4}
\]