1) Evaluate \( \lim_{x \to -1} \frac{x^2-2x-3}{x^2-1} \). If the limit doesn't exist write DNE.

\[
\lim_{x \to -1} \frac{x^2-2x-3}{x^2-1} = \lim_{x \to -1} \frac{(x+1)(x-3)}{(x+1)(x-1)} = \lim_{x \to -1} \frac{x-3}{x-1} = 2.
\]

2) Find \( \lim_{h \to 0} \frac{1+h}{h} \). If the limit doesn't exist write DNE.

We have that \( \lim_{h \to 0} \frac{1+h}{h} = g'(4) \) where \( g(x) = \frac{1}{x} = x^{-1} \).

By derivative formula, \( g'(x) = -x^{-2} = -\frac{1}{x^2} \). So \( g'(4) = -\frac{1}{16} \).

3) If \( f(x) = \sqrt{x^2 + 2x + 1} \), find the slope of the secant line when \( a = 2 \) and \( h = -0.05 \).

Slope of secant line is given by \( \frac{f(a+h) - f(a)}{h} = \frac{\sqrt{(1.95)^2 + 2(1.95)} + 1}{-0.05} - \frac{\sqrt{9}}{-0.05} = 0.4635 \) (to 4 decimal places).

4) Find \( \lim_{x \to \infty} \frac{2x^3-5x^2+12x}{3x^3+2x-6} \). If the limit doesn't exist write DNE.

Divide top and bottom by \( x^3 \) and we get \( \lim_{x \to \infty} \frac{2 - \frac{5}{3x} + \frac{12}{x^2}}{3 + \frac{2}{x} - \frac{6}{x^3}} = \frac{2}{3} \).

5) Find all the vertical and horizontal asymptotes of the curve \( f(x) = \frac{x^2 - 2x + 20}{8x - 16} \).

a) For \( x = 2 \) the denominator is zero and the numerator is not zero so \( x = 2 \) is a vertical asymptote. It is the only root of the denominator so it is the only vertical asymptote.

b) \( \lim_{x \to \infty} \frac{\sqrt{4x^2 - 3x + 20}}{8x - 16} = \lim_{x \to \infty} \frac{x \sqrt{4 - \frac{3}{x} + \frac{20}{x^2}}}{8x - 16} = \lim_{x \to \infty} \frac{x \sqrt{4 - \frac{3}{x} + \frac{20}{x^2}}}{8x - 16} \).

\[
\lim_{x \to \infty} \frac{\sqrt{4 - \frac{3}{x} + \frac{20}{x^2}}}{\frac{8 - 16}{x}} = \frac{1}{4} \quad \text{So } y = \frac{1}{4} \text{ is a horizontal asymptote.}
\]

c) In the limit as \( x \) goes to \( -\infty \), \( x \) is negative and \( \sqrt{x^2} = -x \). So we get \( \lim_{x \to -\infty} \frac{-x \sqrt{4 - \frac{3}{x} + \frac{20}{x^2}}}{8x - 16} = -\frac{1}{4} \) and \( y = -\frac{1}{4} \) is another vert. asymptote.
6) A dynamite blast blows a rock straight up into the air and a formula for its height, in feet, after \( t \) seconds is given by the formula \( h = 160 - 16t^2 \). Find its velocity after 2 seconds (i.e. rate of change of the height with respect to time, \( \frac{dh}{dt} \)).

velocity, \( v = \frac{dh}{dt} = 160 - 32t \). So when \( t = 2 \), \( v = 96 \) ft/s.

7) Find the equation of the tangent line to the curve \( y = 2 \sqrt{x} \)

at the point \((1, 2)\).

\[ y = 2 \sqrt{x} = 2x^{\frac{1}{2}} \]

Then \( \frac{dy}{dx} = x^{-\frac{1}{2}} \) and when \( x = 1 \) \( \frac{dy}{dx} = 1 \).

Then tangent line is \( (y - 2) = (x - 1) \) or \( y = x + 1 \).

8) At which point(s) will the tangent line to the curve \( y = x^3 - 6x^2 + 10 \) have a slope \(-9\)?

The slope at any point \( x \) would be given by \( \frac{dy}{dx} = 3x^2 - 12x \). To find the point we are looking for we need to solve \( 3x^2 - 12x = -9 \) or \( 3x^2 - 12x + 9 = 0 \). \( 3x^2 - 12x + 9 = 3(x-1)(x-3) = 0 \) gives us the points \((1, 5)\) and \((3, -17)\).

9) Find \( \lim_{x \to -3} \frac{x^2 + 3x + 3}{x^3 - x^2 - 12x} \). If the limit doesn't exist write \( DNE \).

\( \frac{x^2 + 3x + 3}{x^3 - x^2 - 12x} = \frac{(x+3)(x+1)}{x(x+3)(x-4)} \).

Hence \( \lim_{x \to -3} \frac{x^2 + 3x + 3}{x^3 - x^2 - 12x} = \lim_{x \to -3} \frac{(x+1)}{x(x-4)} = -\frac{2}{21} \).

10) The limit below represents the derivative of some function \( f(x) \) at some point \( x = a \). \textit{What is the function} \( f(x) \) \textit{and what is} \( a \)?

\( f'(a) = \lim_{h \to 0} \frac{(2+h)^3 - 8}{h} \)

\( \textbf{Answer:} \quad f(x) = x^3 \quad \text{and} \quad a = 2 \)

11) Find \( \lim_{x \to -2} \frac{x-1}{x+2} = \frac{-3}{0^-} = -(-\infty) = \infty \).

(Note: When \( x \) is to the left of \(-2\), then \( x+2 \) is negative. That is why the denominator is approaching \( 0^- \) as \( x \to -2^- \))

Note: \( \lim_{x \to -2} \frac{x-1}{x+2} = -\frac{3}{0^+} = -\infty \). Since \( \lim_{x \to -2} \frac{x-1}{x+2} \) is not equal to \( \lim_{x \to -2} \frac{-1}{x+2} \) we have that \( \lim_{x \to -2} \frac{x-1}{x+2} \) \( DNE \).

\textit{Keep in mind} that if \( \lim_{x \to a} f(x) \) exists then \( \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) \).
12.) Find the point on the parabola \( y = 2x^2 - 4x + 18 \) for which the tangent line does not cross the x-axis.

An equation for a line is \( y = mx + b \) where \( m \) is the slope. If \( m \) is not zero then when \( x = -\frac{b}{m} \) we get \( y = 0 \). That means the line crosses the x-axis at \( -\frac{b}{m}, 0 \). Hence the only way a line will not cross the x-axis is when \( m = 0 \) (i.e. horizontal line). We need to see when \( m = \frac{dy}{dx} = 4x - 4 = 0 \). Only point is \( (1, 16) \).

13.) If \( g(t) = t^3 - \frac{1}{\sqrt{t^4}} \), then find \( g'(t) \).

\[ g(t) = t^3 - t^{-4/3} \]

So \( g'(t) = 3t^2 - \frac{4}{3}t^{-7/3} \).

14.) Suppose gas is leaking from a container and the amount of gas left after \( t \) days is given by the equation \( A(t) = 100e^{-0.18t} \) cu.ft.

a) How much gas did we start with? and

b) How many days will it take to be left with 50 cu.ft.?

a) Question is what \( A(0) \) and that equals \( 100e^0 = 100 \).

b) Solve for \( 50 = 100e^{-0.18t} \) or \( \frac{1}{2} = e^{-0.18t} \). This can be rewritten as

\[-0.18t = \ln\left(\frac{1}{2}\right) = -\ln(2) \]

So \( t = \frac{\ln(2)}{0.18} = 3.85 \). So answer is, **4 days**.

15.) For \( f(x) = \frac{1}{x+2} \) use formula \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) to compute \( f'(x) \).

\[
\begin{align*}
\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \\
&= \lim_{h \to 0} \frac{(x+2) - (x+h+2)}{(x+h+2)(x+2)} \\
&= -\frac{1}{(x+2)^2}.
\end{align*}
\]

(\textit{Note:} After doing it you could check your answer from the quotient rule.)

16.) Find the inverse function \( f^{-1}(x) \) for \( f(x) = \frac{x-2}{2x+1} \).

\( y(2x + 1) = 2yx + y = x - 2 \); \( x(1 - 2y) = y + 2 \).

\( \text{Then } x = \frac{y+2}{1-2y} \) and \( f^{-1}(x) = \frac{x+2}{1-2x} \).

\( \text{(Quick check: } f^{-1}(1) = \frac{1+2}{1-4} = \frac{3}{-3} = -1) \)
17) Given that \( e^{x^2} \cdot e^{-2x+1} = 1 \), solve for \( x \).

\[ e^{x^2-2x+1} = 1. \] This gives us \( x^2 - 2x + 1 = (x - 1)^2 = 0. \)

Conclusion is that \( x = 1 \).

18) If \( f(x) = \left( \frac{x^2+1}{2x+1} \right) \left( \frac{x^4-1}{x^2} \right) \), find a formula for \( f'(x) \).

By multiplying we get \( f(x) = \frac{x^6+3x^4-3x^2-3}{2x^3+x^2} \). Now use quotient rule.

\[ f'(x) = \frac{(6x^5+12x^3-6x)(2x^3+x^2) - (x^6+3x^4-3x^2-3)(6x^2+2x)}{(2x^3+x^2)^2} \]

**You can leave it in this form**

19) For \( x > 0 \), find the domain of the function \( y = \sqrt{x^2-x} \).

We need \( x^2 - x = x(x - 1) \geq 0 \). That means that \( x \geq 1 \).

(Note: If it didn't say \( x > 0 \) then we would also have \( x \leq 0 \) in the domain.)

20) Suppose \( f \) and \( g \) are differentiable functions at \( x = 1 \).

Given that \( f(1) = 2 \), \( f'(1) = 0 \), \( g(1) = 5 \) and \( g'(1) = -1 \),

Then at \( x = 1 \) find the values of:

\[ \frac{d}{dx}(f \cdot g) \quad \text{and} \quad \frac{d}{dx}\left( \frac{f}{g} \right). \]

\[ \frac{d}{dx}(f \cdot g) = f'(1)g(1) + f(1)g'(1) = (0)(5) + (2)(-1) = -2 \]

(product rule)

\[ \frac{d}{dx}\left( \frac{f}{g} \right) = \frac{f'(1)g(1) - f(1)g'(1)}{g(1)^2} = \frac{2}{25} \]

(quotient rule)