9. *) $(e^{2x}+y-1)dx - 1\,dy = 0$

$M = e^{2x}+y-1 \Rightarrow M_y = 1$; $N = -1 \Rightarrow N_x = 0$

SO $M_y \neq N_x$ AND *) IS NOT EXACT. HOWEVER,

$\frac{N_x - M_y}{N} = \frac{0-1}{-1} = 1 = g(x)$. THUS CAN FIND INT. FCTR.

BY SOLVING $\mu' + g\mu = 0 \Rightarrow \mu' + \mu = 0 \Rightarrow e^{-x} = \mu$

$\mu \cdot *)$: $(e^{x} + ye^{-x} - e^{-x})dx - e^{-x}dy = 0$ IS EXACT.

USE EXACT EQ'N ALGOR., GET $y = Ce^{x} + e^{2x} + 1$. NOW

USE I.C. $2 = y(0) = C+1+1 \Rightarrow C = 0 \Rightarrow \boxed{y = e^{2x}+1}$

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10. *) $\frac{dy}{dt} = -y\ln y$. let $u = \ln y$, so

$\frac{du}{dt} = \frac{1}{y}\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = y\frac{du}{dt}$ AND *) $y\frac{du}{dt} = -y\ln y = -yu$

ASSUME $y \neq 0$, + *) $\frac{du}{dt} = -u \Rightarrow u(t) = Ce^{-t}$

$\Rightarrow \boxed{\ln y = Ce^{-t}}$, THIS IS AN IMPLICIT SOL'N OF *).

USE THE I.C. $y(1) = 2$, so $\ln 2 = Ce^{-1} \Rightarrow C = (\ln 2)e$

THEN $\ln y = (\ln 2)ee^{-t} \Rightarrow \ln y = (\ln 2)e^{1-t}$, or

$y = e^{(\ln 2)e^{1-t}} = (e^{\ln 2})^{e^{1-t}} = \boxed{2^{e^{1-t}}}$