PS3 SOLNS

9. *) $x^2 y'' - 6xy' = 0$

IS EULER (FORM $x^2 y'' + \alpha x y' + \beta = 0$ WITH $\alpha = -6$, $\beta = 0$. *IND) IS $r^2 + (\alpha - 1) r + \beta = 0$

OR, $r^2 - 7r = 0 \Rightarrow r_1 = 0$, $r_2 = \Rightarrow y = Ax^0 + Bx^7$

$$\boxed{y = A + Bx^7}$$

(OR, REDUCE ORDER BY LETTING $u = y'$)

10. *) $4x^2 y'' + 4xy' - y = 0$, $y(1) = 0$, $y'(1) = 1$

DIV BY 4:

*) $x^2 y'' + xy' - \frac{1}{4} y = 0$, " , "

THIS IS AN EULER EQN, $\alpha = 1$, $\beta = -\frac{1}{4}$

*IND) $r^2 + (\alpha - 1) r + \beta = 0$.

$r^2 - \frac{1}{4} = 0 \Rightarrow r = \pm \frac{1}{2}$

$\therefore y(x) = A x^{1/2} + B x^{-1/2}$

SO $y'(x) = \frac{1}{2} A x^{-1/2} - \frac{1}{2} B x^{-3/2}$

USE I.C. $0 = y(1) = A \cdot 1 + B \cdot 1 = A + B$

$1 = y'(1) = \frac{1}{2} A \cdot 1 - \frac{1}{2} B \cdot 1 = \frac{1}{2} A - \frac{1}{2} B$

OR: $A + B = 0$, $A - B = 2 \Rightarrow A = 1$, $B = -1$

$$\boxed{y(x) = \sqrt{x} - \frac{1}{\sqrt{x}}}$$