

PROBLEM SET #4. DUE 3/20 AT 4:30 IN CUPPLES I, ROOM 100

1. $x^2 y'' + (x^2 - 2x)y' + 2y = 0$ has a regular singular point at $x = 0$. Hence it has a solution $y(x) = x^r \sum_{n=0}^{\infty} a_n x^n$, where r is the larger of the two roots of the indicial equation. Find this solution. Then express it in terms of elementary functions without using series.

2. For the equation of problem 1, show that there is *not* a solution of the form

$$y(x) = x^r \sum_{n=0}^{\infty} a_n x^n, \text{ where } r \text{ is the } \textit{smaller} \text{ of the two roots.}$$

Note: In the problems below, $L\{f(t)\}$ or $F(s)$ denotes the Laplace transform of $f(t)$.

3. Show that $L\{\sinh t\} = \frac{1}{s^2 - 1}$.

4. Show that the integral which would define $L\{t^t\}$ does not converge.

5. Solve the IVP $y''' - y = 0$, $y(0) = y'(0) = 0$, $y''(0) = 1$.

6. Let $f(t) = \begin{cases} 0 & \text{if } t < 1 \\ t^2 & \text{otherwise} \end{cases}$. Assume $t > 0$. Find $L\{f(t)\}$.

7. Solve $y''' + y' = e^x$, $y(0) = y'(0) = y''(0) = 0$.

8. Let $f(t) = \begin{cases} 0 & t < 1 \\ 2 & t \geq 1 \end{cases}$. Solve $y'' + y = f(t)$, $y(0) = y'(0) = 0$.

9. A mass of 1 Kg is suspended at equilibrium from a spring of Hooke's constant 1. At $t = c$, it is hit by an instantaneous impulse force of k N. Find the function $y(t) =$ displacement from equilibrium at time t . If $k = 3$ and $c = \pi/2$, what is the maximum value of $y(t)$?

10. Suppose that the system of problem 9 is now struck instead by a unit impulse at $t = \pi$ and by another unit impulse force at $t = 2\pi$. Express $y(t) = f(t) \sin t$ for some f . Graph $y(t)$.