10. $*) \begin{cases} x' = -y & x(0) = 1 \\ y' = x & y(0) = 0. \end{cases}$

a) APPLY $\mathcal{L}$ TO $*)$:

$$\mathcal{L}* \begin{cases} sX - 1 = -Y \\ sY = X \end{cases}$$

SOLVE THIS FOR $X, Y \longrightarrow X = \frac{s}{s^2+1},\ Y = \frac{1}{s^2+1}$

so $x(t) = \mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} = \cos t$

$y(t) = \mathcal{L}^{-1}\left\{\frac{1}{s^2+1}\right\} = \sin t$

$$\therefore \boxed{\vec{x}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}}$$

b) $\vec{x}' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\vec{x} \Rightarrow$ EVALS ARE $r = \pm i$, $i \longleftrightarrow \vec{\xi} = \begin{pmatrix} i \\ 1 \end{pmatrix}$

so $\vec{x} = \begin{pmatrix} i \\ 1 \end{pmatrix} e^{it} = \begin{pmatrix} i \\ 1 \end{pmatrix}(\cos t + i \sin t) = \begin{pmatrix} i\cos t - \sin t \\ \cos t + i \sin t \end{pmatrix}$

$= \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} + i \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

HENCE G.S. IS $\vec{x}(t) = c_1 \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$

AND $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \vec{x}(0) = c_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} c_1 = 0 \\ c_2 = 1 \end{matrix}$

so soln is $\boxed{\vec{x}(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}}$