

Math 217 Problem set #6. Due 4:30 PM, Monday 12/1

1. Find the general solution of  $\vec{x}' = \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix} \vec{x}$ . Find and graph the specific solutions defined by  $\vec{x}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$  and  $\vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ . Show direction in which the trajectories are swept out.
2. Solve  $\vec{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} e^{-2t} \\ -2e^t \end{pmatrix}$  using undetermined coefficients.
3. Solve the system of problem 2 using variation of parameters.
4. Find the G.S. of  $\vec{x}' = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \vec{x} + \begin{pmatrix} 0 \\ \sec t \end{pmatrix}$ , where  $0 \leq t \leq \pi/2$ .
5. Let \*)  $\vec{x}' = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix} \vec{x}$  for  $a \neq 0$ . Classify the critical point  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$  as to type and stability.
6. Find all critical points of the system  $\vec{x}' = \begin{pmatrix} 1 & 2 \\ -1 & -2 \end{pmatrix} \vec{x}$ .
7.  $\vec{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \vec{x} + \begin{pmatrix} 1 \\ 2 \end{pmatrix}$  has just one critical point. Find it, its type and stability.
8. Find all critical points with type and stability for  $\begin{cases} x' = xy - 2x \\ y' = xy + y \end{cases}$ .
9. Find and classify all critical points of  $\begin{cases} x' = 2x - x^2 - xy \\ y' = 3y - 2y^2 - xy \end{cases}$ .
10. Draw a phase plane portrait of the system in problem 9, sketching the behavior of the trajectories near each of the critical points.