PROBLEM SET #1. DUE 1/30 at 4:30 PM in Cu I Room 100

Be sure to show your work.

1. Draw a direction field for the equation $y' = y - y^2$. Identify the isoclines and any equilibrium solutions, indicating whether the latter are stable or unstable.

2. Same, for the equation $y' = y^2 - y$.

3. Find the general solution of $\frac{dy}{dt} + y = t^2$. How do the solutions y(t) behave as $t \to \infty$?

4. Equations of the form $y' = ay - by^3$, where a and b are positive constants, arise in the study of fluid dynamics. Solve the IVP: $y' = y - y^3$, $y(0) = 1$.

5. Solve the IVP: $\frac{dy}{dx} = (1-x)y^2$, $y(0) = 2$. The solution is defined on an interval $(-\infty, a)$. Find a.

6. (Baseball physics) A model for the velocity of a batted baseball is : $dv/dt = -rv$, $dw/dt = -g - rw$, where r is the coefficient of air friction, g is the acceleration due to gravity, and v and w are respectively the x and y components of the velocity. Use this model, with r = 0.2 and g = 32 ft/sec^2 to solve the following problem. A baseball is hit straight toward the left field fence with an initial velocity of 150 ft/sec, inclined at an angle of 30 degrees above the horizontal. When hit, the ball is 3 feet above the ground. The left field fence is 10 feet tall, and lies 350 feet from the plate. Does the ball clear the fence for a home run?

7. Refer to problem 6 of sec'n 2.3 of the text and use Toricelli's principle to answer the following. A tank in the form of a right circular cylinder has a radius of 1 m and a height of 5 m. In the bottom is a circular hole 1 cm in radius. If the tank is filled to the top with water, how long does it take to empty?

8. (Refer to problem 19 in sec'n 2.5) In the situation of problem 7, suppose water also enters the top of the tank at a rate of k m^3/sec. Is there an equilibrium depth to the water? If there is, is the equilibrium stable or unstable?

9. Solve the IVP: $y' = e^2 x + y - 1$, $y(0) = 2$ for y(x).

10. Use the substitution $u = \ln y$ to solve $dy/dt = -y \ln y$, $y(1) = 2$. Equations of this general type are used in population modeling.