Solutions

HOMEWORK 4, MATH 233
DUE TUESDAY, SEPTEMBER 24, 2002

Problems 3 is worth 2 points. The other 3 problems are worth 1 point each for a total of 5 points.

(1) Use the polar command in Matlab to first graph the curve with polar equation $r = 1+\cos(5\theta)$ over the interval $[0, 2\pi]$ and next graph the curve with polar equation $r = \sin(\theta/4)$ over the interval $[0, 8\pi]$. Do these graphs separately, i.e. don't put both on the same plot.

(2) Let $f(x, y) = xy/(x^2 + y^2)$ for $(x, y) \neq (0, 0)$.
   (a) Use Matlab to plot the graph of f with a meshgrid for which both x and y go from -.95 to .95 in increments of .1. Note that this means the meshgrid doesn't contain $(0, 0)$.
   (b) Show by hand that $f(x, y)$ is constant on the straight line through the origin with slope m. In terms of m, what is this constant value? Which value for m gives the maximum value of f?
   (c) Use (b) to explain the bad behaviour of your graph for (x, y) near $(0, 0)$.

(3) Do #50 on page A67, Appendix H.1. First find the two points at which the circles intersect, then use the formula for slopes of polar curves to show that their tangent lines are perpendicular at each point of intersection.

(4) Do#24 on page A72, Appendix H.2.

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%Script for #2 on HW#4
[x,y]=meshgrid(-.95:.1:.95);
surf(x,y,x.*y./(x.^2+y.^2));
title('ENW, Graph of f(x,y)=xy/(x^2+y^2)')
xlabel('x','FontSize',14);
ylabel('y','FontSize',14);
zlabel('z','FontSize',14)

%Script for #3a on HW#4
t=0:pi/30:2*pi
polar(t,1+4*cos(5*t))
title('ENW, Graph of the polar equation r=1+r cos5\theta')

%Script for #3b on HW#4
t=0:pi/60:8*pi;
polar(t,sin(t./4));
title('ENW, Graph of the polar equation r=sin(\theta/4) over [0,8\pi]')
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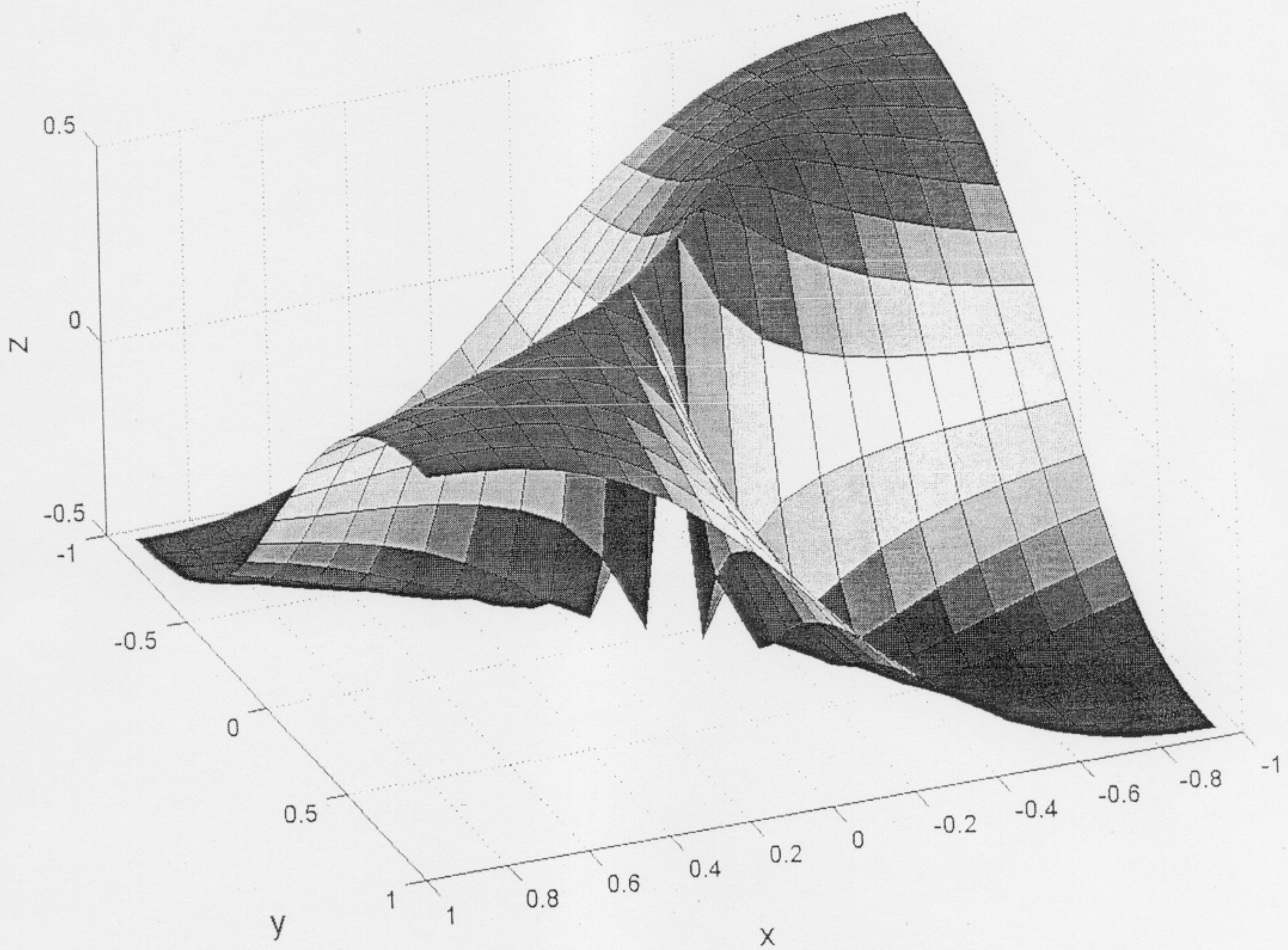
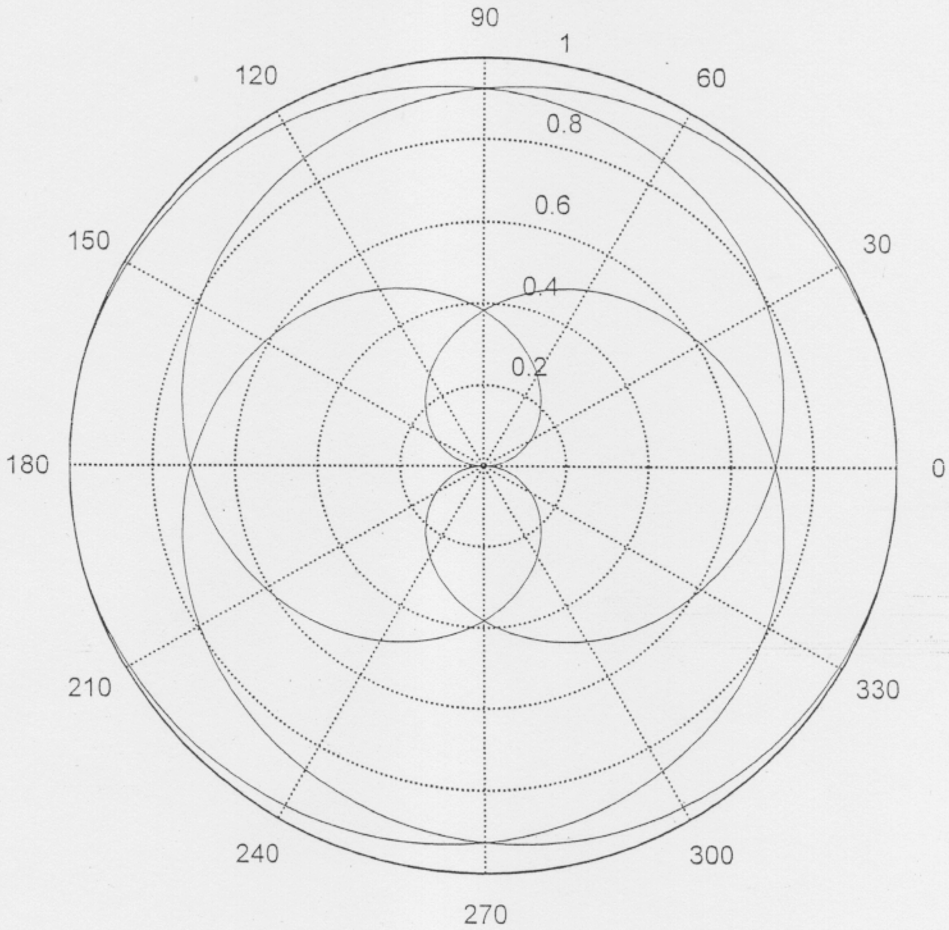
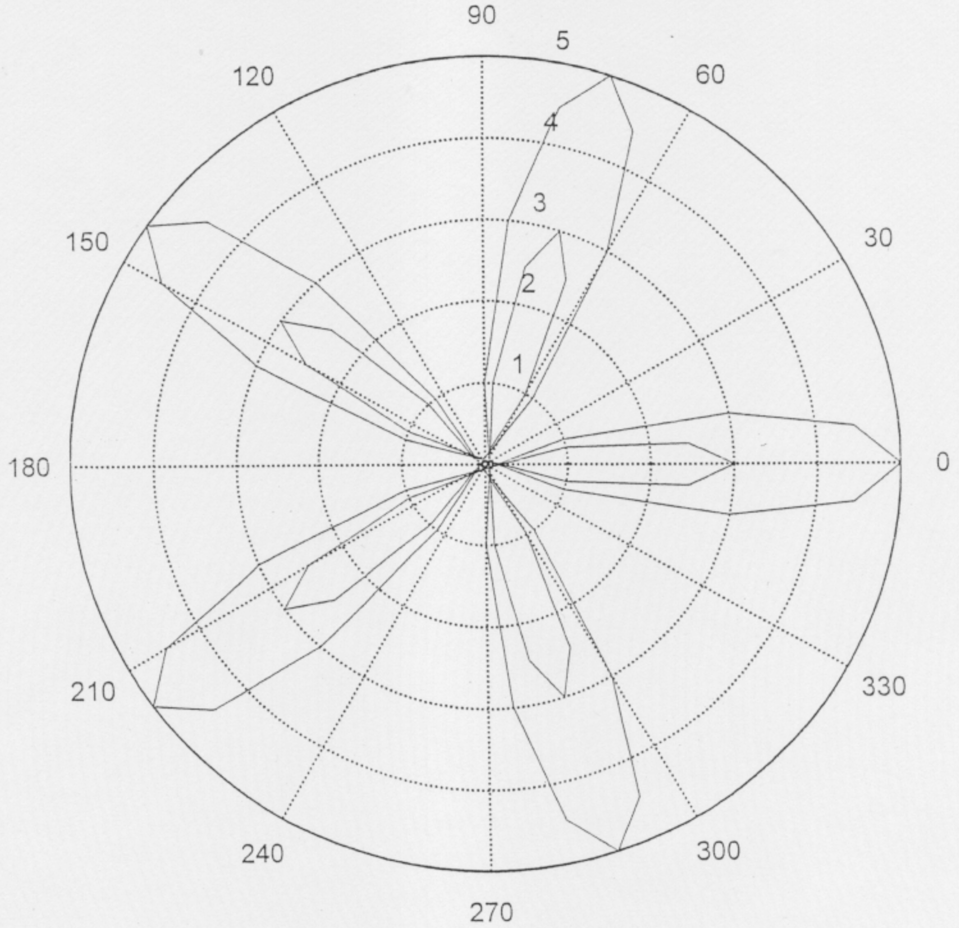
ENW, Graph of $f(x,y)=xy/(x^2+y^2)$



ENW, Graph of the polar equation r=sin(θ/4) over [0,8π]



ENW, Graph of the polar equation r=1+r cos5θ



Solutions - HW #4

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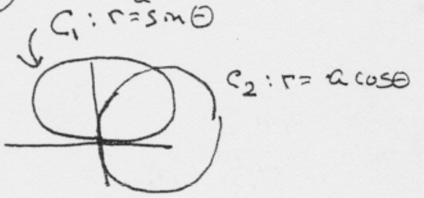


Let C_1 and C_2 be the circles with equations $r = a\sin\theta$ and $r = a\cos\theta$.

For C_1,
$$y = r\sin\theta = a\sin^2\theta$$
$$x = r\cos\theta = a\sin\theta\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2a\sin\theta\cos\theta}{a(\cos^2\theta - \sin^2\theta)}$$

For C_2,
$$y = r\sin\theta = a\sin\theta\cos\theta$$
$$x = r\cos\theta = a\cos^2\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a(\cos^2\theta - \sin^2\theta)}{-2a\cos\theta\sin\theta}$$

We see from this that the product of the slopes is -1 for every θ. Actually the curves have only 2 intersection points

(i) <u>Origin</u> : $\theta = 0$ or π for C_1, $r = 0$
$\theta = \pm\pi/2$ for C_2, $r = 0$

For C_1, the slope $\frac{dy}{dx} = 0$ at the origin while for C_2, the slope $\frac{dy}{dx}$ is infinite at the origin

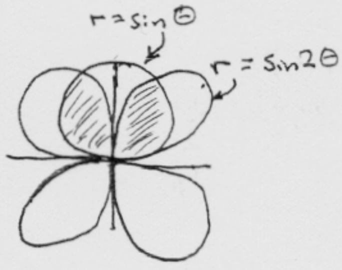
The two ~~tangents~~ tangent lines are the x & y axes and these are perpendicular

(ii) $\theta = \pi/4$, $r = a\sqrt{2}/2$ for both curves

Here dy/dx is infinite for C_1 and is 0 for C_2

The 2 tangent lines are the horizontal and vertical lines through the intersection point. Again, they are perpendicular

4 #24, p. A72, App H.2



The shaded region shows the points inside both the circle $r = \sin\Theta$ and the 4 leaf clover $r = \sin 2\Theta$

The curves intersect at the origin and when $\sin\Theta = \sin 2\theta = 2\sin\Theta\cos\Theta$ i.e. $2\cos\Theta = 1$ and $\Theta = \pm \pi/3$. By symmetry, the total area is twice the area in the first quadrant.

$$A = 2\left\{ \int_0^{\pi/3} \tfrac{1}{2}\sin^2\theta \, d\Theta + \int_{\pi/3}^{\pi/2} \tfrac{1}{2}\sin^2 2\theta \, d\theta \right\}$$

$$= \int_0^{\pi/3} \frac{1-\cos 2\Theta}{2}\, d\Theta + \int_{\pi/3}^{\pi/2} \frac{1-\cos 4\theta}{2}\, d\theta$$

$$= \frac{\Theta}{2}\Big|_0^{\pi/2} - \frac{\sin 2\Theta}{4}\Big|_0^{\pi/3} - \frac{\sin 4\theta}{8}\Big|_{\pi/3}^{\pi/2}$$

$$= \pi/4 - \frac{\sin 2\pi/3}{4} + \frac{\sin 4\pi/3}{8}$$

$$= \pi/4 - \frac{\sqrt{3}}{2}\left(1/4 + 1/8\right)$$

$$= \boxed{\pi/4 - \frac{3\sqrt{3}}{16}}$$