DO THE FOLLOWING PROBLEMS FROM THE TEXTBOOK:

p. 68 #7, #12, #13, #21 (for #7, note that what Conway calls the trace of $\gamma$ is the graph of the function $s = t \sin(1/t)$ for $t \in [0, 1]$)

pp. 74-75 #7, #8, #9(a)-(e), #10, #11

[In case you're using an edition of Conway's book with different page numbering, the above problems are taken from the lists of problems following sections IV.1 and IV.2]

Remarks: For "general interest", you may want to rapidly skim Conway's lengthy discussion of rectifiable curves and Riemann–Stieltjes integrals over such curves. As mentioned in class, none of this is needed for complex analysis. For the above problems involving integrals over closed curves, don't use admissible parametrizations but, instead, use our class discussion of Cauchy's Theorem (and the Cauchy integral formulas) to just "read off" the answer.
Additional Problem (2 point problem): Generalize Problem 8, p. 75 in the following way. Suppose we have a bounded domain $D_1 \subset \mathbb{C}$ and a conformal map $F$ from $D_1$ onto a possibly unbounded domain $D_2 \subset \overline{\mathbb{C}}$.

(i) Use $F$ to define a $1 - 1$ linear map $F^*$ : $\text{Hol}(D_2) \rightarrow \text{Hol}(D_1)$ for which $\int_{F(C)} f \ d\zeta = \int_C F^* f \ d\zeta$ for each closed, piecewise smooth curve $C \subset D_1$ and each $f \in \text{Hol}(D_2)$. [For the reasons mentioned in class, the fact that we could get the same equality using closed, rectifiable curves is of no interest.]

(ii) Show that $F^*$ maps $\text{Hol}(D_2)$ onto $\text{Hol}(D_1)$ $\Leftrightarrow F$ is 1-1. Illustrate this for the case $D_1 = D_2 = \mathbb{D} \setminus \{0\}$ with $\zeta = F(z) = z^n$ for some integer $n \geq 2$ by identifying explicitly the image under $F^*$ of $\text{Hol}(\mathbb{D} \setminus \{0\})$.

(iii) Now suppose $\partial D_1$ is a simple, piecewise smooth, closed curve. Let $H(\overline{D_1})$ be the space of functions which are continuous on $\overline{D_1}$ and holomorphic on $D_1$; this is often described as the subspace of $\text{Hol}(D_1)$ whose members have continuous boundary values. What is the inverse image under $F^*$ of $H(\overline{D_1})$ in $\text{Hol}(D_2)$? Does the equality in (i) still hold for $C=\partial D_1$? Illustrate your conclusions for the case when $D_1 = \mathbb{D}$, $D_2 = \mathbb{H}$ (standard upper half plane), and $F$ is the inverse Cayley transform from $\mathbb{D}$ onto $\mathbb{H}$.

Remark: As time permits, later in the year we'll generalize the above problem by considering, for $1 \leq p < \infty$, the Banach space $H^p(\mathbb{D}) = \text{subspace of } \text{Hol}(\mathbb{D}) \text{ whose members have } L^p \text{ ($p^{th}$ power integrable) boundary values on } \partial \mathbb{D} = C(0,1)$ and
investigating the subspace of $\text{Hol}(\mathbb{H})$ which corresponds to $\text{H}^p(\mathbb{D})$ via the Cayley transform and its inverse.