ASSIGNMENT #8 (DUE MONDAY, NOVEMBER 9)

DO THE FOLLOWING PROBLEMS:

1. # 2(c), 2(d), 2(e) on p. 117 in the textbook (these are part of the Exercises following Section 6.2 on Residues)

2. #6 on p. 118

3. #8 on p. 118

4. #9 on p. 118

5. Recall that \( \hat{\phi}(\xi) = \int_{\mathbb{R}} \phi(x)e^{-2\pi i \xi x} \, dx \) defines the Fourier transform \( \hat{\phi} \) at \( \xi \) of a function \( \phi: \mathbb{R} \rightarrow \mathbb{C} \) when \( \phi \) is integrable on \( \mathbb{R} \).

   For \( x + iy \) in the upper half plane \( \mathbb{H} \), let \( P_y(x) = \frac{y}{\pi(x^2+y^2)} \) and \( Q_y(x) = \frac{x}{\pi(x^2+y^2)} \). As we'll discuss later, these two functions (called the Poisson kernel and conjugate Poisson kernel for \( \mathbb{H} \)) play a very important role in the construction of harmonic functions on \( \mathbb{H} \) with designated bounded values on \( \mathbb{R} \) and certain properties of the construction rest on the values of their Fourier transforms. Use residues to calculate \( \hat{P}_y(\xi) \) and \( \hat{Q}_y(\xi) \) for each \( y > 0 \) and \( \xi \in \mathbb{R} \) and check that they have the same magnitudes. Caution: You'll need to use the limit as
$R \to \infty$ of either upper semi – circular contours or lower semi – circular contours of radius $R$ about 0 depending on the sign of $\xi$. Also be careful applying the residue theorem to these contours.