

Math 350 - Midterm test - March 5, 2010

1. If X and Y have a joint probability density function given by

$$f(x, y) = 2e^{-(x+2y)}$$

for x and y in $(0, \infty)$, find the probability $P(X < Y)$.

2. Let X be a binomial random variable X with parameters (n, p) . Explain why

$$P\left\{\frac{X - np}{\sqrt{np(1-p)}} \leq x\right\} \approx \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-s^2/2} ds$$

for large values of n .

3. Suppose that X and Y are independent, identically distributed, exponential random variables with parameter λ . Let U and V be random variables such that $U = X$ and $V = X + Y$. Find the joint probability density function $g(u, v)$ of U and V . (Don't forget to also indicate where $g(u, v)$ is zero.)

4. If $x_0 = 2$ and

$$x_n \equiv 3x_{n-1} \pmod{7}$$

find x_1, x_2, x_3, x_4 . (Pick them from the set $\{0, 1, \dots, 6\}$.)

5. Explain in a clear and succinct way how the Monte Carlo method can be used to approximate the value of the integral

$$I = \int_0^1 \int_0^1 e^{(x+y)^2} dx dy.$$