

Math 350 - Homework 1

Due 1/29/2010

The probabilistic experiment consisting of picking a random number between 0 and 1 with the uniform probability distribution over the interval $[0, 1]$ is approximately realized on a computer by generating a pseudo-random number (typically by using some kind of *multiplicative congruential method* to be discussed later). In Matlab, the expression

```
>> r=rand
```

returns such a number r . More generally, the command

```
>> r=rand(m,n)
```

returns a matrix with m rows and n columns whose entries are independent uniformly distributed (pseudo-) random numbers between 0 and 1. For example,

```
>> r=rand(1,4)
```

$r =$

```
0.6324    0.0975    0.2785    0.5469
```

Answer the following questions:

1. By experimenting with Matlab, describe concisely in words what each of the following expressions does. (An expression of the form (*something*) has value 0 or 1 depending on whether *something* is False or True.) For example, (b) below can be described as: “the random variable b is 1 if r lies in the interval $(1/3, 2/3]$ and b is 0 if r lies outside that interval.” In (d), also explain briefly why d and b are always equal regardless of the value of r .
 - (a) $r = \text{rand}; a = (r \leq 1/2)$
 - (b) $r = \text{rand}; b = (r > 1/3 \ \& \ r \leq 2/3)$
 - (c) $r = \text{rand}; c = (r \leq 1/3 \ | \ r > 2/3)$
 - (d) $r = \text{rand}; d = (r > 1/3) * (r \leq 2/3)$
2. Let us agree to represent “Heads” by 1 and “Tail” by 0. Then the expression $c = (\text{rand}(1,5) < 1/2)$ may be regarded as simulating the experiment of tossing a fair coin 5 times. The *frequency* of Heads is the ratio of the number of Heads over the total number of tosses and is calculated by $\text{sum}(c)/5$. Now do the following numerical experiment: Toss a fair coin 100 times and calculate the frequency of Heads in the sample.

3. Consider the following experiment: Toss a fair coin 10 times and count the number of Heads. The result is a random variable X taking values in the set $\{0, 1, \dots, 10\}$. Write a short Matlab script that does the following: repeat the experiment 1000 times to obtain a sequence of independent random variables $Y = (X_1, X_2, \dots, X_{1000})$, where each X_j is an integer between 0 and 10. Now plot a histogram of Y with 10 bins centered at the numbers $0, \dots, 10$, using the command `hist(Y, 0 : 10)`. (Use the Matlab help facility to learn more about histograms.) In a few words, explain what the x -axis and y -axis of the histogram represent.
4. Using `rand` as above, show how to simulate the experiment of tossing a biased coin with probability $1/3$ of Heads and $2/3$ of Tail.
5. Similarly, show how to simulate the experiment of tossing a fair die. I.e., simulate a random variable that takes values in $\{1, 2, 3, 4, 5, 6\}$, each outcome having probability $1/6$.