Math 350 - Homework 1 - Solutions

1. Interpret the following Matlab expressions:

   (a) $\texttt{r} = \texttt{rand}; \texttt{a} = (\texttt{r} <= 1/2)$

   The random variable a takes only the values 1 or 0. It is 1 if and only if r is less than or equal to $1/2$, so $a = 1$ with probability $1/2$ and 0 with probability $1/2$.

   (b) $\texttt{r} = \texttt{rand}; \texttt{b} = (\texttt{r} > 1/3 \ \& \ \texttt{r} <= 2/3)$

   The random variable b takes values 1 or 0. It is one if and only if r is both (strictly) greater than $1/3$ and less than or equal to $2/3$. Therefore, $b = 1$ happens with probability $1/3$ and it is 0 with probability $2/3$.

   (c) $\texttt{r} = \texttt{rand}; \texttt{c} = (\texttt{r} <= 1/3 \mid \texttt{r} > 2/3)$

   The random variable c takes only the values 1 or 0. It is 1 if and only if r is either less than or equal to $1/3$ or r is (strictly) greater than $2/3$. Note that $c = 1 - b$. It is 1 with probability $2/3$ and 0 with probability $1/3$.

   (d) $\texttt{r} = \texttt{rand}; \texttt{d} = (\texttt{r} > 1/3) * (\texttt{r} <= 2/3)$

   The random variable d is the product of two random variables which can only take the values 0 or 1. So d can only be 0 or 1. It is 1 if and only if both $(r > 1/3) = 1$ and $(r <= 2/3) = 1$, i.e., if and only if both $r > 1/3$ and $r \leq 2/3$ are true. Therefore, d is exactly the same random variable as b.

2. Do the following numerical experiment: Toss a fair coin 100 times and calculate the frequency of Heads in the sample.

   The array $\texttt{c} = (\texttt{rand}(1, 100) < 1/2)$; is a sequence of 100 random numbers from the set $\{0, 1\}$, each chosen so that 1 has probability $1/2$. The frequency of 1s is then given by $\texttt{f} = \texttt{sum}(\texttt{c})/100$. More concisely, you may simply calculate the value of $\texttt{f} = \texttt{sum}((\texttt{rand}(1, 100) < 1/2))/100$. Typical values of f that you may obtain are 0.52, 0.49, 0.55, etc. Naturally, we expect this to be approximately 50%.

3. Consider the following experiment: Toss a fair coin 10 times and count the number of Heads. The result is a random variable X taking values in the set $\{0, 1, \ldots, 10\}$. Write a short Matlab script that does the following: repeat the experiment 1000 times to obtain a sequence of independent random variables $Y = (X_1, X_2, \ldots, X_{1000})$, where each X_j is an integer between 0 and 10. Now plot a histogram of Y with 10 bins centered at the numbers $0, \ldots, 10$, using the command $\texttt{hist}(\texttt{Y}, 0 : 10)$. (Use the Matlab help facility to learn more about histograms.) In a few words, explain what the x-axis and y-axis of the histogram represent.

   The random variable $\texttt{X} = \texttt{sum}(\texttt{rand}(1, 10) < 1/2)$ gives the number of Heads in one trial of the experiment. One way to perform 1000 trials is to use a `for` statement as shown below. I suggest that you write an m.file rather than do this directly on the command window. Go to the File menu at the top of the Mathlab command window and open a blank m.file. Then type something like (anything that follows a percentage sign is simply a comment, and is not involved in the execution of the script):

```
%Let Y be the vector whose entries are the outcomes of each
%trial of the experiment of counting the number of heads
%after tossing 10 fair independent coins. The entries of Y
%are numbers from {0,1, ... , 10}.
Y=zeros(1,1000); %Initialize Y as a vector with entries equal to 0.
for i=1:1000
    Y(1,i)=sum(rand(1,10)<1/2);
end
%To visualize the outcome, we plot a histogram of Y having
%10 bins, each centered at one of the integers 0, 1, ... , 10.
hist(Y,0:10)
```

Then run the script by clicking on an icon for this purpose at the top of the editor window. (The icon looks like a little triangle pointing to the right.) You can also write the scrip using any other text editor and then paste it on the command window.

The outcome is the graph shown below:

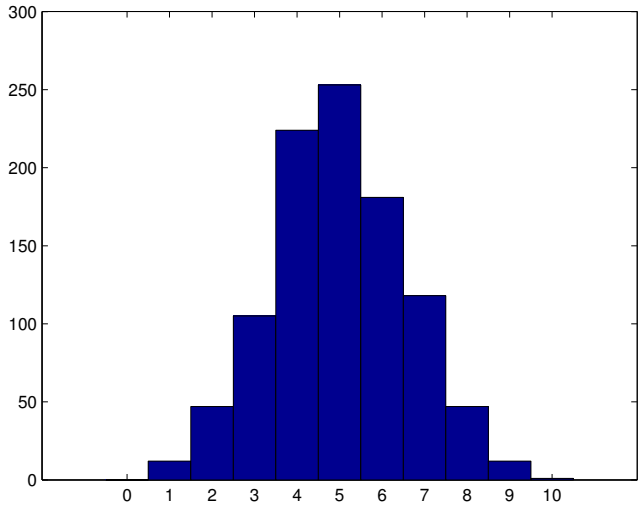

Figure 1: Histogram for exercise 3.

The x-axis gives the centers of each bin and the y-axis gives the occupation number. For example, the tallest bar, over $x = 5$, is the number of trials of the experiment in which the outcome was 5. It occurred about 250 times out of 1000 trials. The least frequent outcome was 0 with no occurrences.

4. Using `rand` as above, show how to simulate the experiment of tossing a biased coin with probability 1/3 of Heads and 2/3 of Tail.

   For a single trial of the experiment simply calculate $\mathtt{c} = (\mathtt{rand}(1,1)) < 1/3$. Then $c = 1$ (i.e., Heads) with probability 1/3.

5. Similarly, show how to simulate the experiment of tossing a fair die. I.e., simulate a random variable that takes values in $\{1, 2, 3, 4, 5, 6\}$, each outcome having probability 1/6.

   One way is to write directly on the command window:

```
>> r=rand;
>> D=1*(r<1/6)+2*(r>=1/6 & r<2/6)+3*(r>=2/6 & r<3/6)...
+4*(r>= 3/6 & r<4/6)+5*(r>=4/6 & r<5/6)+6*(r>=5/6)
```

Another way is to write a function file such as:

```
 function [D] = DieExperiment(n)
%The input n is the number of independent tosses of the die.
%The output is a vector of dimension n, where each entry is
%a random number from {1, 2, ... , 6} with probability 1/6.
r=rand(1,n);
D=1*(r<1/6)+2*(r>=1/6).*(r<2/6)+3*(r>=2/6).*(r<3/6)...
    +4*(r>=3/6).*(r<4/6)+5*(r>=4/6).*(r<5/6)+6*(r>=5/6);
end
```

This should be written in a Matlab editor window and saved as an m.file. The name of the file must be the same as the name of the function itself. In my example it would be `DieExperiment.m`. Once this file is saved, you can use the function as the following example.

```
 >> D=DieExperiment(10)

D =

     3     5     4     2     4     1     4     5     3     5
```