

Math 350 - Homework 4

Due 2/19/2010

1. (Text, problem 1, page 46.) If $x_0 = 5$ and

$$x_n = 3x_{n-1} \pmod{150}$$

find x_1, \dots, x_{10} .

2. (Text, problem 6, page 47.) This problem refers to the integral

$$I = \int_0^{\infty} x(1+x^2)^{-2} dx.$$

- (a) Find the exact value of I .
(b) Estimate the error made by approximating I by

$$I_R = \int_0^R x(1+x^2)^{-2} dx.$$

How big should R be if we wish $|I - I_R|$ to be less than, say, 10^{-2} ?

- (c) Use Monte Carlo simulation to approximate the integral and compare your estimate with the exact value. I suggest taking $R = 20$.
3. (Text, problem 8, page 47.) Use Monte Carlo to approximate the value of the double integral

$$\int_0^1 \int_0^1 e^{(x+y)^2} dx dy.$$

4. (Text, problem 10, page 47.) Use simulation to approximate $\text{Cov}(U, e^U)$, where U is uniform on $(0, 1)$. Compare your approximation with the exact answer.
5. (Text, problem 13, page 47.) For independent uniform $(0, 1)$ random variables U_1, U_2, \dots define

$$N = \text{Minimum} \left\{ n : \sum_{i=1}^n U_i > 1 \right\}.$$

That is, N is equal to the number of random numbers that must be summed to exceed 1.

- (a) Estimate $E[N]$ by generating 100 values of N .
(b) Estimate $E[N]$ by generating 1000 values of N .
(c) Estimate $E[N]$ by generating 10000 values of N .
(d) What do you think is the value of $E[N]$?

As always, I plan to discuss some of these problems in class during the week. It will help a lot if you think about them beforehand.