1. (Text, problem 1, page 87, modified.) We wish to generate a random variable $X$ taking values in $[0,1]$, having probability density function

$$f(x) = e^x/(e - 1).$$

(a) Write a program based on the inverse transform method to generate $X$.

(b) Show that your program works as desired by doing the following: obtain a graph of the approximate density function using a suitably normalized histogram plot from a large number of simulated values of $X$; then superpose the graph of the exact function $f(x)$ to the first graph.

2. (Text, problem 7, page 88. The composition method.) Suppose it is relatively easy to generate random variables from any of the distributions $F_i$ for $i = 1, \ldots, n$. How could we generate a random variable having the distribution

$$F(x) = \sum_{i=1}^{n} p_i F_i(x)$$

where $p_i$, $i = 1, \ldots, n$, are nonnegative numbers whose sum is 1?

3. (Text, problem 8 (a), page 88.) Using the result of Exercise 7, give an algorithm for generating random variables from the cumulative distribution function

$$F(x) = \frac{x + x^3 + x^5}{3}, \quad 0 \leq x \leq 1.$$

4. (Text, problem 10, page 89.) A casualty insurance company has 1000 policy holders, each of whom will independently present a claim in the next month with probability 0.05. Assuming that the amounts of the claims made are independent exponential random variables with mean $800, use simulation to estimate the probability that the sum of these claims exceeds $50,000.

5. (Text, problem 24, page 91.) Buses arrive at a sporting event according to a Poisson process with a rate 5 per hour. Each bus is equally likely to contain either 20, 21, \ldots, 40 fans, with the numbers in the different buses being independent. Write an algorithm to simulate the arrival of fans to the event by time $t = 1$.

As always, I plan to discuss some of these problems in class during the week. It will help a lot if you think about them beforehand.