Math 350 - Homework 6

Due 3/19/2010

1. (Text, problem 1, page 87, modified.) We wish to generate a random variable X taking values in $[0, 1]$, having probability density function
$$f(x) = e^x/(e-1).$$
   (a) Write a program based on the inverse transform method to generate X.
   (b) Show that your program works as desired by doing the following: obtain a graph of the approximate density function using a suitably normalized histogram plot from a large number of simulated values of X; then superpose the graph of the exact function $f(x)$ to the first graph.

2. (Text, problem 7, page 88. The composition method.) Suppose it is relatively easy to generate random variables from any of the distributions F_i $i = 1, \dots, n$. How could we generate a random variable having the distribution
$$F(x) = \sum_{i=1}^{n} p_i F_i(x)$$
where p_i, $i = 1, \dots, n$, are nonnegative numbers whose sum is 1?

3. (Text, problem 8 (a), page 88.) Using the result of Exercise 7, give an algorithm for generating random variables from the cumulative distribution function
$$F(x) = \frac{x + x^3 + x^5}{3}, \; 0 \le x \le 1.$$

4. (Text, problem 10, page 89.) A casualty insurance company has 1000 policy holders, each of whom will independently present a claim in the next month with probability 0.05. Assuming that the amounts of the claims made are independent exponential random variables with mean \$800, use simulation to estimate the probability that the sum of these claims exceeds \$50,000.

5. (Text, problem 24, page 91.) Buses arrive at a sporting event according to a Poisson process with a rate 5 per hour. Each bus is equally likely to contain either $20, 21, \dots, 40$ fans, with the numbers in the different buses being independent. Write an algorithm to simulate the arrival of fans to the event by time $t = 1$.

   As always, I plan to discuss some of these problems in class during the week. It will help a lot if you think about them beforehand.