

Math 350 - Homework 8

Due 4/5/2010

1. (Text, problem 1, page 133.) For any set of numbers x_1, \dots, x_n , prove algebraically that

$$\sum_{i=1}^n (x_i - \bar{x})^2 = \sum_{i=1}^n x_i^2 - n\bar{x}^2$$

where $\bar{x} = \sum_{i=1}^n x_i/n$.

2. (Text, problem 3, page 133.) Write a program that uses the recursions given by Equations (7.6) and (7.7) to calculate the sample mean and sample variance of a data set.
3. (Text, problem 6, page 133.) Estimate $\int_0^1 \exp(x^2) dx$ by generating random numbers. Generate at least 100 values and stop when the standard deviation of your estimator is less than 0.01.
4. (Text, problem 8, page 134.) It can be shown that if we add random numbers until their sum exceeds 1, then the expected number added is equal to e . That is

$$N = \min \left\{ n : \sum_{i=1}^n U_i > 1 \right\}$$

then $E[N] = e$.

- (a) Use this idea to estimate e , using 1000 simulation runs.
- (b) Estimate the variance of the estimator in (a) and give a 95 percent confidence interval estimate of e .
5. (Text, problem 10, page 134.) Use the approach that is presented in Example 3a of Chapter 3 to obtain an interval of size less than 0.1, which we can assert, with 95 percent confidence, contains π . How many runs were necessary?

As always, I plan to discuss some of these problems in class during the week. It will help a lot if you think about them beforehand.