

# Math 350 - Homework 9

Due 4/12/2010

1. (Text, problem 1, page 209, modified.) Suppose we wanted to estimate  $\theta$ , where

$$\theta = \int_0^1 e^{x^2} dx.$$

- (a) Show that  $Y_1 = e^{U^2}(1 + e^{1-2U})/2$  is an unbiased estimator of  $\theta$ ;  
(b) Show that  $Y_1$  is a better estimator than  $Y_2 = (\exp(U_1^2) + \exp(U_2^2))/2$ , where  $U_1$  and  $U_2$  are independent random numbers.
2. (Text, problem 2, page 210.) Explain how antithetic variables can be used in obtaining a simulation estimate of the quantity

$$\theta = \int_0^1 \int_0^1 e^{(x+y)^2} dx dy.$$

Is it clear in this case that using antithetic variables is more efficient than generating a new pair of random variables?

3. (Text, problem 4, page 210.) Show that if  $X$  and  $Y$  have the same distribution then  $\text{Var}[(X + Y)/2] \leq \text{Var}(X)$ , and conclude that the use of antithetic variables can never increase variance (although it need not be as efficient as generating an independent set of random numbers).
4. (Text, problem 6, page 210.) Suppose that  $X$  is an exponential random variable with mean 1. Give another random variable that is negatively correlated with  $X$  and that is also exponential with mean 1.
5. (Text, problem 5, page 210.)
- (a) If  $Z$  is a standard normal random variable, design a study using antithetic variables to estimate  $\theta = E[Z^3 e^Z]$ .
- (b) Using the above, do the simulation to obtain an interval of length no greater than 0.1 that you can assert, with 95% confidence, contains the value of  $\theta$ .

**As always, I plan to discuss some of these problems in class during the week. It will help a lot if you think about them beforehand.**