Math 350 - Homework 10

Due 4/16/2010

1. (Text, problem 10, page 211.) In certain situations a random variable X, whose mean is known, is simulated so as to obtain an estimate of $P\{X \leq a\}$ for a given constant a. The raw simulation estimator from a single run is I, where
$$I = \begin{cases} 1 & \text{if } X \leq a \\ 0 & \text{if } X > a. \end{cases}$$
Because I and X are clearly negatively correlated, a natural attempt to reduce the variance is to use X as a control—and so use an estimator of the form $I + c(X - E[X])$.
   (a) Determine the percentage of variance reduction over the raw estimator I that is possible (by using the best c) if X were uniform on $(0, 1)$.
   (b) Repeat (a) if X were exponential with mean 1.
   (c) Explain why we knew that I and X were negatively correlated.

2. (Text, problem 11, page 211.) Show that $\mathrm{Var}(\alpha X + (1-\alpha)W)$ is minimized by α being equal to the value given in Equation (8.3) and determine the resulting variance.

3. (Text, problem 12, page 211.)
   (a) Explain how control variables may be used to estimate θ in Exercise 1. (The latter is Problem 1 of homework assignment 9.)
   (b) Do 100 simulation runs, using the control given in (a), to estimate first c^* and then the variance of the estimator.
   (c) Using the same data as in (b), determine the variance of the antithetic variable estimator.
   (d) Which of the two types of variance reduction techniques worked better in this example?

4. (Text, problem 15, page 211.) Show that in estimating $\theta = E[(1 - U^2)^{1/2}]$ it is better to use U^2 rather than U as the control variate. to do this, use simulation to approximate the necessary covariances.

As always, I plan to discuss some of these problems in class during the week. It will help a lot if you think about them beforehand.