

# Math 449 Fall 2006 - Midterm

10/18/06

1. Let  $f(x) = x^2 - x - 3$ .
  - (a) Find the Newton-Raphson formula  $x_k = g(x_{k-1})$ .
  - (b) Start with  $x_0 = 0$  and find  $x_1$  and  $x_2$ .
  - (c) Suppose that the bisection method is used to approximate the zero of  $f(x)$  in the interval  $[2, 3]$ . How many times must this interval be bisected to guarantee that the approximation  $c_n$  has accuracy of  $10^{-10}$ ?

2. Find the triangular factorization  $A = LU$  for

$$A = \begin{pmatrix} 1 & -2 \\ 4 & 2 \end{pmatrix}$$

3. Consider the following linear system of equations

$$\begin{aligned} 4x - y &= 15 \\ x + 5y &= 9. \end{aligned}$$

- (a) Write down the Gauss-Seidel iteration formula for this system.
  - (b) Starting with  $\mathbf{P}_0 = \mathbf{0}$ , use this iteration to find  $\mathbf{P}_1$  and  $\mathbf{P}_2$ .
  - (c) Does the iteration converge? (Why?)
4. Compute the divided-difference table for the tabulated function and write down the Newton polynomial  $P_2(x)$ .

$k$	$x_k$	$f(x_k)$
0	4.0	2.00
1	5.0	2.24
2	6.0	2.45

5. Let  $f(x) = x + \frac{2}{x}$ . Use quadratic Lagrange interpolation based on nodes  $x_0 = 1$ ,  $x_1 = 2$ , and  $x_2 = 2.5$  to approximate  $f(1.5)$ .
6. Let  $g(x) = 1 + \frac{1}{2x}$ .
- (a) Start with the initial point  $x_0 = 1$ . Compute the points  $x_1$  and  $x_2$  in the fixed point iteration of  $g(x)$ .
  - (b) Explain graphically how the iteration proceeds. (Draw your graphs carefully enough to be able to decide visually whether the iteration converges.)
  - (c) How can we determine analytically if the iteration converges? (Hint: consider the derivative of  $g(x)$  for  $x \geq 1$ .)