

# Math 5031 - Homework 1

Due 9/9/05

1. Show that if every element  $a$  in a group  $G$  has order 2, that is,  $a^2 = e$ , then  $G$  is an abelian group.
2. Assume that the equation  $abc = e$  holds for some  $a, b, c$  in a group  $G$ . Does it follow that  $bca = e$ ? How about  $bac = e$ ?
3. Show that every group of order less than or equal to 5 is abelian.
4. Let  $G$  be a group. A *commutator* in  $G$  is an element of the form  $aba^{-1}b^{-1}$ , with  $a, b \in G$ . Let  $G^c$  be the subgroup generated by the commutators, which is called the *commutator subgroup* of  $G$ . Show that  $G^c$  is a normal subgroup. Show that any homomorphism of  $G$  into an abelian group factors through  $G/G^c$ .
5. Let  $G$  be a group such that  $\text{Aut}(G)$  is cyclic. Prove that  $G$  is abelian.
6. Let  $G$  be a finite group of even order. Show that  $G$  contains an element of order 2.