Math 5031 - Homework 4

Due 9/30/05

1. Prove that every group of order $|G| < 60$ is solvable.

The following results can be freely used. (Some will have been shown in class but possibly not all. Make sure that you can prove them, although you do not need to write down the proofs for this assignment.)

Proposition 1 *Let p and q be distinct prime numbers and G a group of order n, where n is one of the following: (i) p^k for some positive integer k, (ii) pq, (iii) p^2q, or (iv) $2pq$. (In the last case, assume that p and q are odd.) Then G is a solvable group.*

Proposition 2 *Let G be a group of order $p_1^{e_1} \dots p_t^{e_t}$, where p_i are distinct primes and e_i are positive integers. Let r_i denote the number of p_i-Sylow subgroups of G. Then r_i divides $|G|/p_i^{e_i}$ and r_i is congruent to 1 modulo p_i.*

Proposition 3 *Let G be a group and H a normal subgroup of G. Then G is solvable if and only if both H and G/H are solvable groups.*

Proposition 4 *Let G be a group of order $p^e m$, where p is a prime, p does not divide m, and p^e does not divide $(m-1)!$. Then G contains a proper normal subgroup.*