1. Prove that every group of order $|G| < 60$ is solvable.

The following results can be freely used. (Some will have been shown in class but possibly not all. Make sure that you can prove them, although you do not need to write down the proofs for this assignment.)

**Proposition 1** Let $p$ and $q$ be distinct prime numbers and $G$ a group of order $n$, where $n$ is one of the following: (i) $p^k$ for some positive integer $k$, (ii) $pq$, (iii) $p^2q$, or (iv) $2pq$. (In the last case, assume that $p$ and $q$ are odd.) Then $G$ is a solvable group.

**Proposition 2** Let $G$ be a group of order $p_1^{e_1} \ldots p_t^{e_t}$, where $p_i$ are distinct primes and $e_i$ are positive integers. Let $r_i$ denote the number of $p_i$-Sylow subgroups of $G$. Then $r_i$ divides $|G|/p_i^{e_i}$ and $r_i$ is congruent to 1 modulo $p_i$.

**Proposition 3** Let $G$ be a group and $H$ a normal subgroup of $G$. Then $G$ is solvable if and only if both $H$ and $G/H$ are solvable groups.

**Proposition 4** Let $G$ be a group of order $p^e m$, where $p$ is a prime, $p$ does not divide $m$, and $p^e$ does not divide $(m - 1)!$. Then $G$ contains a proper normal subgroup.