

Math 5031 - Homework 9

Due 11/11/05

1. Given the polynomials in $\mathbb{C}[x, y]$:

$$f_1 = x^2 + y^2 - 1, \quad f_2 = x^2 - y + 1, \quad f_3 = xy - 1,$$

prove that there are $p_1, p_2, p_3 \in \mathbb{C}[x, y]$ such that $1 = p_1 f_1 + p_2 f_2 + p_3 f_3$.
(Note: you do not need to find the p_i explicitly.)

2. A variety is said to be *reducible* if it can be expressed as the union of two proper subvarieties; otherwise the variety is called *irreducible*. Show that a variety V is irreducible if and only if its associated ideal $\mathcal{I}(V)$ is prime. (This corresponds to problems 1-4 section 8.1 of Ash's text. There is a link to his lectures on my web page.)
3. (This corresponds to problems 5,6 in the same section.) Show that any variety is the union of finitely many irreducible varieties, and that the decomposition is unique (assuming that we discard any subvariety that is contained in another one).
4. (This corresponds to problems 7, 8.) Suppose that the field of definition is algebraically closed. Show that \mathbb{A}^n is compact in the Zariski topology.