Math 5032 - Homework 2

Due 2/10/06

1. If $f : M \to M$ is an R-module homomorphism such that $f \circ f = f$, show that $M = \mathrm{Ker} f \oplus \mathrm{Im} f$.

2. If $f : M \to N$ and $g : N \to M$ are R-module homomorphisms such that $g \circ f = 1_M$ (the identity homomorphism of M), show that $N = \mathrm{Ker} g \oplus \mathrm{Im} f$.

3. (The Five Lemma). Let

$$\begin{array}{ccccccccc} A_1 & \longrightarrow & A_2 & \longrightarrow & A_3 & \longrightarrow & A_4 & \longrightarrow & A_5 \\ \downarrow{\scriptstyle \alpha_1} & & \downarrow{\scriptstyle \alpha_2} & & \downarrow{\scriptstyle \alpha_3} & & \downarrow{\scriptstyle \alpha_4} & & \downarrow{\scriptstyle \alpha_5} \\ B_1 & \longrightarrow & B_2 & \longrightarrow & B_3 & \longrightarrow & B_4 & \longrightarrow & B_5 \end{array}$$

   be a commutative diagram of R-modules and R-module homomorphisms, with exact rows. Prove that:

   (a) if α_1 is an epimorphism and α_2, α_4 are monomorphisms then α_3 is a monomorphism;

   (b) if α_5 is a monomorphism and α_3, α_4 are epimorphisms then α_3 is an epimorphism.

4. Describe all abelian groups of orders 144 and 168.

5. If p and q are distinct primes describe all abelian groups of order

   (a) p^2q^2

   (b) p^4q

   (c) p^n, $1 \leq n \leq 6$

   (d) p^3q^2.