

Math 5051 - Homework 10

Due 11/13/08

1. (Problem 22, page 100) If $f \neq 0$, there exist $C, R > 0$ such that $Hf(x) \geq C|x|^{-n}$ for $|x| > R$. Hence $m(\{x : Hf(x) > \alpha\}) \geq C'/\alpha$ when α is small, so the estimate in the maximal theorem is essentially sharp.
2. (Problem 24, page 100) If $f \in L^1_{\text{loc}}$ and f is continuous at x , then x is in the Lebesgue set of f .
3. (Problem 25, page 100) If E is a Borel set in \mathbb{R}^n , the *density* $D_E(x)$ of E at x is defined as

$$D_E(x) = \lim_{r \rightarrow 0} \frac{m(E \cap B(r, x))}{m(B(r, x))},$$

whenever the limit exists.

- (a) Show that $D_E(x) = 1$ for a.e. $x \in E$ and $D_E(x) = 0$ for a.e. $x \in E^c$.
 - (b) Find examples of E and x such that $D_E(x)$ is a given number $\alpha \in (0, 1)$, or such that $D_E(x)$ does not exist.
4. (Problem 26, page 100) If λ and μ are positive, mutually singular Borel measures on \mathbb{R}^n and $\lambda + \mu$ is regular, then so are λ and μ .