Math 5051 - Homework 2

Due 9/18/08

1. (Problem 19, page 32) Let μ^* be an outer measure on X induced from a finite premeasure μ_0. If $E \subset X$, define the **inner measure** of E to be

$$\mu_*(E) = \mu_0(X) - \mu^*(E^c).$$

   Then E is μ^*-measurable iff $\mu^*(E) = \mu_*(E)$. (Use Exercise 18, of first homework assignment.)

2. (Problem 30, page 40) If $E \in \mathcal{L}$ and $m(E) > 0$, for any $\alpha < 1$ there is an open interval I such that $m(E \cap I) > \alpha m(I)$.

3. (Problem 3, page 48) If $\{f_n\}$ is a sequence of measurable functions on X, then $\{x : \lim f_n(x) \text{ exists}\}$ is a measurable set.

4. (Problem 8, page 48) If $f : \mathbb{R} \to \mathbb{R}$ is monotone, then f is Borel measurable.

5. (Problem 9, page 48) Let $f : [0,1] \to [0,1]$ be the Cantor function (§1.5), and let $g(x) = f(x) + x$.
   (a) g is a bijection from $[0,1]$ to $[0,2]$, and $h = g^{-1}$ is continuous from $[0,2]$ to $[0,1]$.
   (b) If C is the Cantor set, $m(g(C)) = 1$.
   (c) By Exercise 29 of Chapter 1, $g(C)$ contains a Lebesgue nonmeasurable set A. Let $B = g^{-1}(A)$. Then B is Lebesgue measurable but not Borel.
   (d) There exists a Lebesgue measurable function F and a continuous function G on $\mathbb{R}$ such that $F \circ G$ is not Lebesgue measurable.