Math 5051 - Homework 3

Due 9/25/08

1. (Problem 14, page 52) If $f \in L^+$, let $\lambda(E) = \int_E f\,d\mu$ for $E \in \mathcal{M}$. Then λ is a measure on $\mathcal{M}$, and for any $g \in L^+$, $\int g\,d\lambda = \int fg\,d\mu$. (First suppose that g is simple.)

2. (Problem 15, page 52) If $\{f_n\} \subset L^+$, f_n decreases pointwise to f, and $\int f_1 < \infty$, then $\int f = \lim \int f_n$.

3. (Problem 16, page 52) If $f \in L^+$ and $\int f < \infty$, for every $\epsilon > 0$ there exists $E \in \mathcal{M}$ such that $\mu(E) < \infty$ and $\int_E f > (\int f) - \epsilon$.

4. (Problem 17, page 52) Assume Fatou's lemma and deduce the monotone convergence theorem from it.

5. Let $T : X \to X$ be a measurable mapping of a measure space $(X, \mathcal{M}, \mu)$. We say that T is *measure-preserving* if $\mu(T^{-1}E) = \mu(E)$, for $E \in \mathcal{E}$. Show that T is measure-preserving iff $\int f \circ T\,d\mu = \int f\,d\mu$ for all $f \in L^+$.