Math 5051 - Homework 4

Due 10/02/08

1. (Problem 20, page 59) If $f_n, g_n, f, g \in L^1$, $f_n \to f$ and $g_n \to g$ a.e., $|f_n| \leq g_n$, and $\int g_n \to \int g$, then $\int f_n \to \int f$. (Rework the proof of the dominated convergence theorem.)

2. (Problem 22, page 59) Let μ be counting measure on $\mathbb{N}$. Interpret Fatou's lemma and the monotone and dominated convergence theorems as statements about infinite series.

3. (Problem 26, page 60) If $f \in L^1(m)$ and $F(x) = \int_{-\infty}^{x} f(t)\,dt$, then F is continuous on $\mathbb{R}$.

4. (Problem 28, a, b, page 60) Compute the following limits and justify the calculations:

   (a) $\lim_{n\to\infty} \int_0^\infty (1 + (x/n))^{-n} \sin(x/n)\,dx$.

   (b) $\lim_{n\to\infty} \int_0^1 (1 + nx^2)(1 + x^2)^{-n}\,dx$.

5. (Problem 31, a, b, page 60) Derive the following formulas by expanding part of the integrand into an infinite series and justifying the term-by-term integration. Exercise 29 may be useful.

   (a) For $a > 0$, $\int_{-\infty}^{\infty} e^{-x^2} \cos ax\,dx = \sqrt{\pi} e^{-a^2/4}$.

   (b) For $a > -1$, $\int_0^1 x^a (1 - x)^{-1} \log x\,dx = -\sum_1^\infty (a + k)^{-2}$.