Math 5051 - Homework 5

Due 10/09/08

1. (Problem 32, page 63) Suppose $\mu(X) < \infty$. If f and g are complex-valued measurable functions on X, define

$$\rho(f,g) = \int \frac{|f-g|}{1+|f-g|} d\mu.$$

   Then ρ is a metric on the space of measurable functions if we identify functions that are equal a.e., and $f_n \to f$ with respect to this metric iff $f_n \to f$ in measure.

2. (Problem 34, page 63) Suppose $|f_n| \leq g \in L^1$ and $f_n \to f$ in measure.

   (a) $\int f = \lim \int f_n$.

   (b) $f_n \to f$ in L^1.

3. (Problem 40, page 63) In Egoroff's theorem, the hypothesis "$\mu(X) < \infty$" can be replaced by "$|f_n| \leq g$" for all n, where $g \in L^1(\mu)$.

4. (Problem 41, page 63) If μ is σ-finite and $f_n \to f$ a.e., there exist measurable $E_1, E_2, \dots \subset X$ such that $\mu((\cup_1^\infty E_j)^c) = 0$ and $f_n \to f$ uniformly on each E_j.

5. (Problem 44, page 64 - **Lusin's Theorem**) If $f : [a,b] \to \mathbb{C}$ is Lebesgue measurable and $\epsilon > 0$, there is a compact set $E \subset [a,b]$ such that $\mu(E^c) < \epsilon$ and $f|E$ is continuous. (Use Egoroff's theorem and Theorem 2.26.)