Math 5051 - Homework 8

Due 10/30/08

1. (Problem 2, page 88) If ν is a signed measure, E is ν-null iff $|\nu|(E) = 0$. Also, if ν and μ are signed measures, $\nu \perp \mu$ iff $\nu^+ \perp \mu$ and $\nu^- \perp \mu$.

2. (Problem 3, page 88) Let ν be a signed measure on $(X, \mathcal{M})$.
   (a) $L^1(\nu) = L^1(|\nu|)$.
   (b) If $f \in L^1(\nu)$, $\left|\int f\, d\nu\right| \leq \int |f|\, d|\nu|$.
   (c) If $E \in \mathcal{M}$, $|\nu|(E) = \sup\{\left|\int_E f\, d\nu\right| : |f| \leq 1\}$.

3. (Problem 6, page 88) Suppose $\nu(E) = \int f\, d\mu$ where μ is a positive measure and f is an extended μ-integrable function. Describe the Hahn decomposition of ν and the positive, negative, and total variations of ν in terms of f and μ.

4. Suppose that ν is a signed measure on $(X, \mathcal{M})$ and $E \in \mathcal{M}$.
   (a) $\nu^+(E) = \sup\{\nu(E) : F \in \mathcal{M}, F \subset E\}$ and $\nu^-(E) = -\inf\{\nu(E) : F \in \mathcal{M}, F \subset E\}$.
   (b) $|\nu|(E) = \sup\{\sum_1^n |\nu(E_j)| : n \in \mathbb{N}, E_1, \ldots, E_n$ are disjoint, and $\cup_1^n E_j j = E\}$.

5. (Problem 8, page 92) $\nu \ll \mu$ iff $\nu^+ \ll \mu$ and $\nu^- \ll \mu$.