Math 5051 - Homework 9

Due 11/06/08

1. (Problem 12, page 92) For $j = 1, 2$, let μ_j, ν_j be σ-finite measures on $(X_j, \mathcal{M}_j)$ such that $\nu_j \ll \mu_j$. Then $\nu_1 \times \nu_2 \ll \mu_1 \times \mu_2$ and

$$\frac{d(\nu_1 \times \nu_2)}{d(\mu_1 \times \mu_2)}(x_1, x_2) = \frac{d\nu_1}{d\mu_1}(x_1)\frac{d\nu_2}{d\mu_2}(x_2).$$

2. (Problem 16, page 92) Suppose that μ, ν are σ-finite measures on $(X, \mathcal{M})$ with $\nu \ll \mu$, and let $\lambda = \mu + \nu$, If $f = d\nu/d\lambda$, then $0 \le f < 1$ μ-a.e. and $d\nu/d\mu = f/(1 - f)$.

3. (Problem 17, page 93) Let $(X, \mathcal{M}, \mu)$ be a finite measure space, $\mathcal{N}$ a sub-σ-algebra of $\mathcal{M}$, and $\nu = \mu|_{\mathcal{N}}$. If $f \in L^1(\mu)$, there exists $g \in L^1(\nu)$ (thus g is $\mathcal{N}$-measurable) such that $\int_E f\, d\mu = \int_E g\, d\nu$ for all $E \in \mathcal{N}$; if g' is another such function, then $g = g'$ ν-a.e. (In probability theory, g is called the **conditional expectation** of f in $\mathcal{N}$.)

4. (Problem 19, page 94) If ν , μ are complex measures and λ is a positive measure, then $\nu \perp \mu$ iff $|\nu| \perp |\mu|$, and $\nu \ll \lambda$ iff $|\nu| \ll \lambda$.

5. (Problem 20, page 94) If ν is a complex measure on $(X, \mathcal{M})$ and $\nu(X) = |\nu|(X)$, then $\nu = |\nu|$.